OPTIMAL PLACEMENT OF PMUs FOR ENHANCING POWER SYSTEM STATE ESTIMATION INCLUDING BOTH V & I PHASORS

BY

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Dedicated to My Beloved Parents.

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LIST OF ABBREVIATIONS

EMS	:	Energy Management System
IRLS	:	Iteratively Reweighted Least Squares
OPP	:	Optimal PMU Placement
PMU	:	Phasor Measurement Unit
PSSE	:	Power System State Estimation
RTU	:	Remote Terminal Unit
SCADA	:	Supervisory Control And Data Acquisition
WAMS	:	Wide Area Monitoring System
WLS	:	Weighted Least Squares

ABSTRACT

Full Name : Huthaifa Mahmoud Abedalaziz Hussein

Thesis Title : Optimal Placement of PMUs for Enhancing Power System State Estimation Including Both V & I Phasors

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Traditional state estimation of power system is based on measurements collected from supervisory control and data acquisition (SCADA) system. Such measurements can give erroneous results in the presence of bad data. Alternatively, phasor measurement units (PMUs), which are synchronized by GPS and have high sampling frequency, provide real-time highly accurate readings of bus-phasor's voltages and branch-phasor's currents. Due to their high cost, however, they will not replace conventional state estimator meters. Rather, they will improve the estimator performance by installing a certain number into a power system. This research investigates an optimal placement of PMUs to a power system with existing conventional meters. A heuristic optimization technique is developed to find the optimum number as well as the best location of both voltage and current PMUs to enhance the accuracy of state estimation. Two algorithms are considered for state estimation evaluation, namely, weighted least squares (WLS) and iteratively reweighted least squares (IRLS). This work modifies the formulation of these two estimators to account for PMU measurements. The performance of these estimators is tested – with and without PMUs – in the presence of different types of bad data using IEEE 14-, 30-, and 118-bus systems. It was found that incorporation of PMUs enhances the performance of both estimators, and IRLS outperforms WLS in rejecting bad data, suppressing bad leverage points, and providing better estimation results.

ملخص الرسالة

الاسم الكامل: حذيفة محمود عبد العزيز حسين

عنوان الرسالة: التوزيع الأمثل لأجهزة قياس الطور من أجل تحسين تقدير الحالة لنُظم القوى مع تضمين طوري الجهد و التيار الكهربائيين

التخصص: الهندسة الكهربائية

تاريخ الدرجة العلمية: يناير ٢٠١٨

تعتمد الأنظمة التقليدية لتقدير حالة أنظمة القوى على جمع قياساتٍ من منظومة التحكم الإشرافي و استحصال البيانات المعروفة اختصاراً بمنظومة (سكادا SCADA). قد تعطي هذه القياسات نتائج خاطئةً عند وجود بياناتٍ سيئةٍ. كبديلٍ عن تلك المنظومة، تزودنا أجهزة قياس الطور (PMUs) – المتزامنة بأقمارٍ اصناعيةٍ عبر نظام تحديد المواقع (GPS) ما يمكنها من تحديث البيانات المجمعة بترددٍ عالٍ – بقياساتٍ آنيةٍ لمطوار جهود العقد الكهربائية و مطوار تيارات الأفرع الكهربائية. و نظراً لارتفاع تكلفة أجهزة المطوار؛ فإنها لن تحل محل المنظومة التقليدية لتقدير حالة النظام. عوضاً عن ذلك، ستحسن أجهزة المطوار من أداء نظام تقدير الحالة بتركيب عددٍ محددٍ منها في نظام الطاقة الكهربائية. يهدف هذا البحث إلى إيجاد توزيعٍ أمثل لأجهزة المطوار في أجهزة الطاقة الموجودة حالياً. واحدةٌ من تقنيات التحسين الأمثل الإرشادية طُوّرت لإيجاد أقل عددٍ ممكن و أفضل موقعٍ لأجهزة قياس طوري الجهد و التيار الكهربائيين من أجل تحسين دقة تقدير حالة النظام. تم اعتبار خوارزميتين لتقييم تقدير الحالة و هما: المربعات الصغرى الموزونة (WLS) و المربعات الصغرى ذات الوزن التكراري (IRLS). يعدل هذا البحث صياغة هذين المقدّرين لحساب قياسات المطوار. تم اختبار أداء كلٍ من مقدِّري الحالة – بتضمين أجهزة قياس الطور و بعدم تضمينها – هذين بوجود أنواعٍ مختلفةٍ من البيانات الخاطئة على أنظمة معهد مهندسي الكهرباء و الإلكترونيات (IEEE) و هي: IEEE-14، IEEE-30، و IEEE-118. و قد وُجد أن دمج وحدات قياس الطور يعزز أداء كلا المقدّرين، و أن أداء المقدر IRLS يفوق أداء المقدر WLS في رفض البيانات السيئة، و كبت نقاط التأثير السيئة، و تقديم نتائج تقديرٍ أفضل.

CHAPTER 1

INTRODUCTION

1.1 Overview of Power System State Estimation

The operating conditions of a power system can be investigated and analyzed by knowing the network model. Voltage phasors at all buses are known as the static states of the power system [1]. To maintain the power system under normal and secure conditions, operators should monitor the system states continuously and take appropriate actions. Before any security assessment can be made or control actions taken, a reliable estimate of the existing state of the system must be determined. This is achieved by collecting measurements from the remote terminal units (RTU) of the supervisory control and data acquisition (SCADA) computers deployed in the transmission system. Such measurements include the on/off status of switching devices such as transformer taps and circuit breakers, as well as real-time data of line flows, output powers of generators and transformers, and voltage magnitudes at most buses. These measurements are then telemetered to the energy management system (EMS) through communication channels. The direct use of the raw measurements is not desired due to many reasons which include measurements error, telemetry failures, communication noise, unavailability of some measurements, … etc. State estimator (SE) at the EMS performs filtering of the received set of measurements to get the best fit of the input data. Figure 1.1 shows the configuration of the EMS/SCADA for a typical power system.

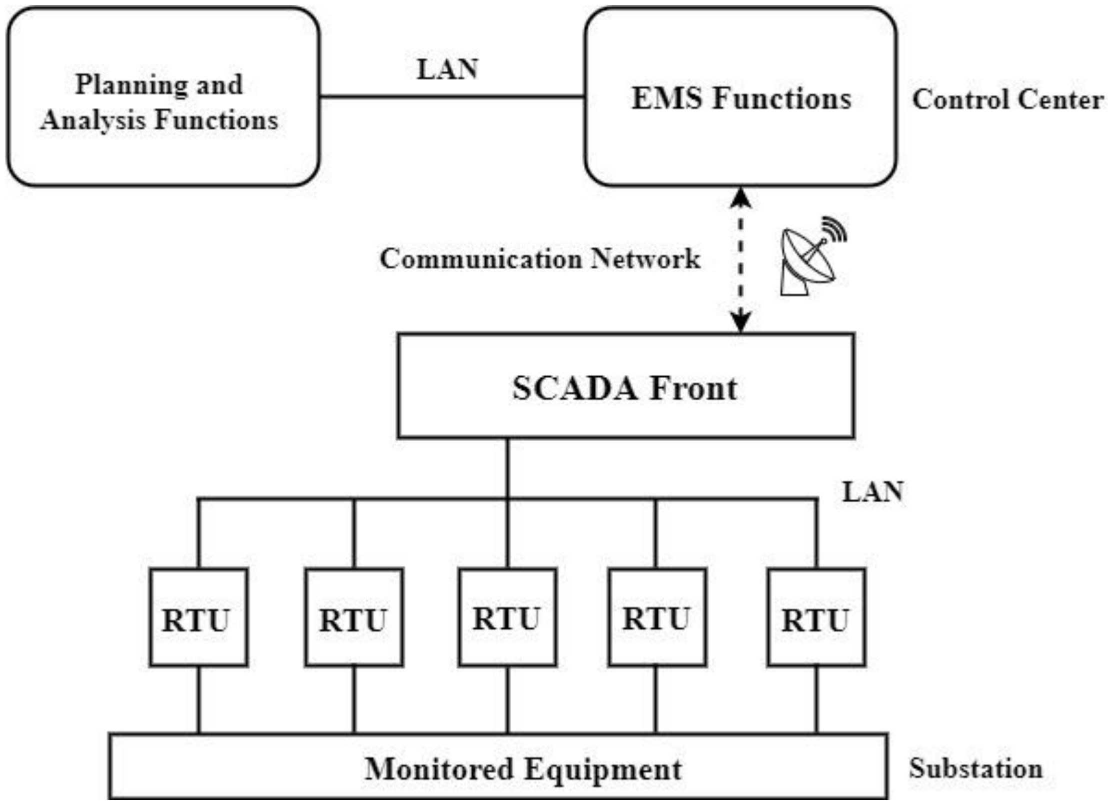

Figure 1.1: EMS/SCADA system configuration.

The state estimation of power system was first proposed by Schweppe in 1968 which he defined as "a data processing algorithm for converting redundant meter readings and other available information into an estimate of the state of an electric power system" [2].

1.2 Motivation of The Thesis

Traditional measurements collected from RTUs of SCADA system do not represent the system accurately during dynamic operations, and bus voltage phase-angles as well as branch current phase-angles are not typically measured [1]. Recently, phasor measurement units (PMUs) are included for the monitoring of power systems. They provide phase angle measurements in addition to the magnitude measurements by synchronizing signals from the global positioning satellite (GPS) which makes these measurements highly accurate.

Since PMUs are expensive devices, an optimization technique is needed to find the best locations and the number of PMUs to be optimally deployed in the power system.

1.3 Thesis Contributions

The main contributions of this thesis are listed as follows:

- Modifying the formulation of two state estimation algorithms: weighted least squares (WLS) and iteratively reweighted least squares (IRLS) by including voltage and current phasors to simulate the incorporation of PMUs.
- Investigating the performance of the estimators in the presence of different bad-data categories with and without PMUs.
- Optimizing the number and locations of PMUs needed to enhance the accuracy of the estimators.
- The performance of an optimized method is tested in the presence of various types of bad data using a heuristic approach.
- IEEE n-bus test systems was used for simulation, specifically, 14-, 30-, and 118-bus systems.

1.4 Thesis Outline

This thesis is divided into five main chapters. The first chapter gives an overview of power system state estimation, motivation, contributions, and outline of the thesis.

The second chapter provides a brief background concerning power system state estimation. Topics reviewed in this chapter generally include estimation algorithms selected in this work, namely weighted least squares (WLS) and iteratively reweighted least squares (IRLS). Issues related to state estimation such as observability and measurement classification are also covered. The chapter concludes with a mathematical model of inclusion current measurements to the state estimator.

Chapter three starts with a short introduction about phasor measurement units (PMUs). After that, it presents the work that has been done in the field of optimal PMU placement. After the literature survey, modeling of PMU measurements is summarized. The chapter ends with a demonstration of the heuristic PMU placement algorithm.

In chapter four, detailed discussion and analysis of the simulation results are presented. In addition, different scenarios are tested, and a comprehensive comparison is provided.

Finally, chapter five gives overall conclusions from the work done and possible future improvements that could be achieved.

CHAPTER 2

BACKGROUND

State estimation is the process of assigning a value to an unknown system state variable based on measurements from that system according to some criteria [3]. In a power system, the state variables are the voltage magnitudes and relative phase angles at the system buses. The estimator is designed to produce the best estimate of the system states recognizing that there are errors in the measured quantities and that there may be redundant measurements. The estimation process is based on a statistical criterion as will be discussed next.

2.1 State Estimation Algorithms

State estimation model is formulated as an over-determined system of nonlinear equations and solved as a weighted least squares (WLS) problem which is explained in [2 – 4] and many subsequent papers. Occasionally, errors may appear in the measurements and in the network structure or parameters. The presence of bad data produces erroneous results since the probability distribution of the measurement error is no longer normal (or Gaussian as assumed). There are two ways of handling bad data and its spreading effect – interacting and noninteracting bad data. The first approach is to use the result of WLS to detect, identify, and remove bad data as discussed in [5]. The second approach is to use a robust algorithm which is less sensitive to small deviations from assumption. Several robust algorithms are reviewed in [6]. One of these robust algorithms is the iteratively reweighted least squares (IRLS) estimator presented in [7].

In this thesis, two algorithms for state estimators will be adopted which are WLS and IRLS. WLS represents the conventional method used widely in power systems whereas IRLS is chosen as a robust estimator to be compared with WLS. The selection of IRLS is based on its advantages over other methods as summarized next.

IRLS algorithm can be implemented through a simple modification of the conventional WLS algorithm and is as fast as WLS [7]. It is a robust estimator which can suppress bad data and bound the influence of bad leverage points [1]. Leverage points are outliers in the space spanned by the row vectors of the Jacobian (or measurement-residual) matrix. They do not conform to the fashion of the majority of the point cloud [7]. IRLS does not downweight good leverage points which enhance the accuracy of M-estimators. An M-estimator is a maximum likelihood estimator which minimizes an objective function of measurement residuals subject to constraints given by the measurement equations [1].

2.1.1 Weighted Least Squares (WLS) Algorithm

For a set of measurements provided by the vector *z*:

$$z = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, \dots,\ x_n) \\ \vdots \\ h_m(x_1, \dots,\ x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e \qquad (2.1)$$

Where:

$z^T = [z_1, \dots, z_m]$: is a $(1 \times m)$ measurement vector.

$x^T = [x_1, \dots, x_n]$: is a $(1 \times n)$ state vector.

$h^T = [h_1(x), \dots, h_m(x)]$

$h_i(x)$: is a nonlinear function mapping measurement i to state vector x – given by power flow equations.

$e^T = [e_1, \ldots, e_m]$: is a ($1 \times m$) measurement error vector.

Measurement errors are assumed to be statistically independent with random normal distribution having a zero mean ($\mu = 0$) and standard deviation σ_i which reflects the accuracy of the meter. This allows us to calculate the weight vector of measurements as follows:

$$cov(e) = E[e \cdot e^T] = R = diag\{\sigma_1^2, \ldots, \sigma_m^2\} = W^{-1} \tag{2.2}$$

The objective function of WLS is to decrease the weighted sum of measurement residuals, that is:

$$\min J(x) = \sum_{i=1}^{m} \frac{[z_i - h_i(x)]^2}{\sigma_i^2} = [z - h(x)]^T R^{-1} [z - h(x)] \tag{2.3}$$

Equating the gradient of the objective function to zero:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \tag{2.4}$$

$$H(x) = \frac{\partial h(x)}{\partial x} \tag{2.5}$$

To linearize equation (2.4), Taylor series expansion to the first order is used to get the iterative equation:

$$\Delta x = x^{k+1} - x^k = -G(x^k)^{-1} \cdot g(x^k) \tag{2.6}$$

$$G(x) = H(x)^T R^{-1} H(x) \tag{2.7}$$

Where k is the iteration index. Matrix G of equation (2.7) is called the Gain matrix which is symmetric and sparse provided the system is fully observable.

The set of Normal equations (2.6) is solved iteratively until reaching an acceptable solution defined by a tolerance value ϵ such that $|\Delta x| \leq \varepsilon$.

Matrix H of equation (2.5) is called the Jacobian matrix which has a dimension of (m×n). For a WLS to be solvable, the system must be overdetermined, that is, the number of measurements (m) is bigger than the number of states (n), or equivalently, H is a full-rank matrix.

2.1.2 Iteratively Reweighted Least Squares (IRLS) Algorithm

In this algorithm, a weight matrix Q is introduced to the Normal equations set (2.6) as follows:

$$\Delta x = [H^T R^{-1} Q H]^{-1} \cdot H^T R^{-1} Q [z - h(x)] \tag{2.8}$$

The method is applied on the Schweppe-Huber Generalized-M (SHGM) estimator which is a robust estimator. The weight matrix Q is diagonal and defined as [6]:

$$Q(i,i) = \begin{cases} 1, & |r_{Si}| \leq c \\ \left|\frac{c}{r_{Si}}\right|, & otherwise \end{cases} \tag{2.9}$$

Where:

Standardized residual: $$r_{Si} = \frac{r_i}{\sigma_i w_i} \tag{2.10}$$

Weight: $$w_i = min\left\{1, \left[\frac{b_i}{PS_i}\right]^2\right\} \tag{2.11}$$

Cutoff Chi-square: $$b_i = \chi^2_{v,0.975} \tag{2.12}$$

Degree of freedom: $$v = no. of\ nonzeros\ in\ H_i \tag{2.13}$$

Projection Statistics: $$PS_i = \max_{\|H_j\|} \frac{\left|H_i^T H_j\right|}{\beta_i}, \quad for\ j = 1, \dots, m \tag{2.14}$$

Robust scale: $$\beta_i = 1.1926 f_m \times \underset{k}{\text{lomed}} \{\underset{j \neq k}{\text{lomed}} \left|H_k^T H_i + H_j^T H_i\right|\} \quad for\ 1 \leq i, j, k \leq m \tag{2.15}$$

The factor f_m is a sample correction factor which makes the scale estimator β unbiased. This factor is defined by (2.16).

m	2	3	4	5	6	7	8	9
f_m	0.743	1.851	0.954	1.351	0.993	1.198	1.005	1.131

(2.16)

For $m > 9$, $f_m = \begin{cases} \frac{m}{m-0.9}, & m\ is\ odd \\ 1, & m\ is\ even \end{cases}$

Lomed in equation (2.15) indicates a low median defined as the [(m+1)/2]-th order statistic, the brackets [x] represents the integer part of x.

Each vector H_i defines a point in the space of regression. Also, these vectors have few nonzero components. The data points that project at the origin on a given direction H_k are irrelevant for H_i in the sense that they do not bring any information about its outlyingness. Therefore, they should be excluded when calculating (2.15) along that direction.

By using the Jacobian matrix processed at a flat voltage, the weights of equation (2.11) – and hence, the projection statistics (2.14) – need to be computed only once, which can be done off-line [7].

The parameter "c" in equation (2.9) is a tuning parameter. A good choice for c is a value ranges between 1 and 3. As c tends to zero, the SHGM estimator reduces to the generalized least absolute value (GLAV) estimator and as c tends to infinity, it becomes the WLS estimator [7]. In this work, a value of 1.5 is chosen for c since it is commonly used in literature.

2.1.3 Non-Linear Function *h(x)* & Jacobian *H*

Conventional measurements consist of: line power flows, bus power injections, and bus voltage magnitudes. When they are expressed in polar coordinates in an N-bus system, the state vector $\boldsymbol{x}$ will have $(2N - 1)$ elements; N bus voltage magnitudes and $(N - 1)$ bus phase angles since the reference bus – usually bus no. 1 – is set to zero, as follows:

$$x^T = [\delta_2, \delta_3, \cdots \delta_N, V_1, V_2, \cdots V_N] \tag{2.17}$$

The expression for each function is given below assuming the general two-port π-model of the network branches shown in Figure 2.1.

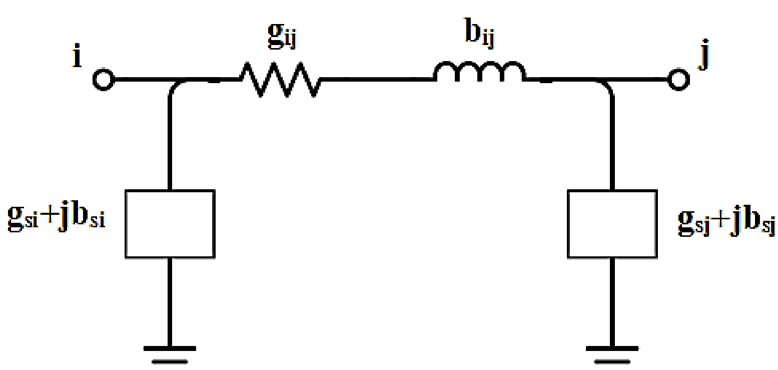

Figure 2.1: Two-port π-model of a network branch.

- Real and reactive power injections at bus i:

$$P_i = V_i \sum_{j \in N_i} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \tag{2.18}$$

$$Q_i = V_i \sum_{j \in N_i} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \tag{2.19}$$

- Real and reactive power flow from bus i to bus j:

$$P_{ij} = V_i^2 (g_{si} + g_{ij}) - V_i V_j (g_{ij} \cos \delta_{ij} + b_{ij} \sin \delta_{ij}) \tag{2.20}$$

$$Q_{ij} = -V_i^2 (b_{si} + b_{ij}) - V_i V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) \tag{2.21}$$

Where: V_i, δ_i is the voltage magnitude and phase angle at bus i, respectively.

$\delta_{ij} = \delta_i - \delta_j$

$G_{ij} + jB_{ij}$ is the ijth element of the complex bus admittance matrix.

$g_{ij} + jb_{ij}$ is the admittance of the series branch connecting buses i and j.

$g_{si} + jb_{si}$ is the admittance of the shunt branch connected at bus i.

The structure of the Jacobian matrix will be as follows:

$$H = \begin{bmatrix} 0 & \frac{\partial V_{mag}}{\partial V} \\ \frac{\partial P_{inj}}{\partial \delta} & \frac{\partial P_{inj}}{\partial V} \\ \frac{\partial Q_{inj}}{\partial \delta} & \frac{\partial Q_{inj}}{\partial V} \\ \frac{\partial P_{flow}}{\partial \delta} & \frac{\partial P_{flow}}{\partial V} \\ \frac{\partial Q_{flow}}{\partial \delta} & \frac{\partial Q_{flow}}{\partial V} \end{bmatrix} \tag{2.22}$$

The expressions for each partition are given below:

- Elements corresponding to real power injections:

$$\frac{dP_i}{d\delta_i} = \sum_{j=1}^{N} V_i V_j (-G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) - V_i^2 B_{ii} \tag{2.23}$$

$$\frac{dP_i}{d\delta_j} = V_i V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \tag{2.24}$$

$$\frac{dP_i}{dV_i} = \sum_{j=1}^{N} V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) + V_i G_{ii} \tag{2.25}$$

$$\frac{dP_i}{dV_j} = V_i (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \tag{2.26}$$

- Elements corresponding to reactive power injections:

$$\frac{dQ_i}{d\delta_i} = \sum_{j=1}^{N} V_i V_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) - V_i^2 G_{ii} \tag{2.27}$$

$$\frac{dQ_i}{d\delta_j} = V_i V_j (-G_{ij} \cos \delta_{ij} - B_{ij} \sin \delta_{ij}) \tag{2.28}$$

$$\frac{dQ_i}{dV_i} = \sum_{j=1}^{N} V_j (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) - V_i B_{ii} \tag{2.29}$$

$$\frac{dQ_i}{dV_j} = V_i (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \tag{2.30}$$

- Elements corresponding to real power flows:

$$\frac{dP_{ij}}{d\delta_i} = V_i V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \tag{2.31}$$

$$\frac{dP_{ij}}{d\delta_j} = -V_i V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \tag{2.32}$$

$$\frac{dP_{ij}}{dV_i} = -V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) + 2V_i (g_{ij} + g_{si}) \tag{2.33}$$

$$\frac{dP_{ij}}{dV_j} = -V_i (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \tag{2.34}$$

- Elements corresponding to reactive power flows:

$$\frac{dQ_{ij}}{d\delta_i} = -V_i V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \tag{2.35}$$

$$\frac{dQ_{ij}}{d\delta_j} = V_i V_j (g_{ij} \cos\delta_{ij} + b_{ij} \sin\delta_{ij}) \tag{2.36}$$

$$\frac{dQ_{ij}}{dV_i} = -V_j (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) - 2V_i (b_{ij} + b_{si}) \tag{2.37}$$

$$\frac{dQ_{ij}}{dV_j} = -V_i (g_{ij} \sin\delta_{ij} - b_{ij} \cos\delta_{ij}) \tag{2.38}$$

- Elements corresponding to voltage magnitudes:

$$\frac{dV_i}{d\delta_i} = 0 \; for \; all \; i \; and \; j, \frac{dV_i}{dV_i} = 1, \frac{dV_i}{dV_j} = 0 \tag{2.39}$$

If voltage phase angles are measured – by PMUs for example – then the Jacobian matrix H (equation 2.22) will have an additional row. The elements corresponding to this additional row:

$$\frac{d\delta_i}{dV_i} = 0 \; for \; all \; i \; and \; j, \frac{d\delta_i}{d\delta_i} = 1, \frac{d\delta_i}{d\delta_j} = 0 \tag{2.40}$$

2.2 State Estimation Considerations

2.2.1 System Observability

State estimation algorithms are based on system measurements collected from various locations. If these measurements are not enough or locations of meters are not appropriate, the estimation problem could not be solved, and the system is said to be unobservable. Observability of a network is specified by the location and type of measurements in addition to the topology of the network.

Given a set of measurements and their locations, the network observability analysis will determine whether a unique estimate can be found for the system state. Observability analysis can be performed on a linearized model of the measurements without loss of generality [1]. The analysis may also be carried out off-line as well as on-line [1].

There are two methods for observability analysis: numerical approach and topological approach. If a system is numerically observable, then it is topologically observable, but not the opposite [8].

Numerical observability refers to the ability of a system model to be solved for a unique state estimate [8]. It is based on the numerical factorization of the Jacobian matrix or the gain matrix. A system is considered numerically observable if the Jacobian matrix is well-conditioned and has a full rank.

Topological observability, on the other hand, refers to the existence of at least one spanning tree with full rank in the network [8]. It is based on the graph theory in which the spanning tree connects all the nodes through branches with measurements assigned to each node.

2.2.2 Measurement Redundancy and Classification

Measurement redundancy is defined as a ratio of number of measurements to number of unknown states [5], is important for observability as well as for bad data identification and detection [9]. Measurements can be classified accordingly into the following [1]:

a. Critical measurement: causes an observable system to become unobservable when it is removed from the measurement set.

b. Redundant measurement: is a non-critical measurement. Only this type of measurement may have a nonzero residual.

c. Critical pair: if the simultaneous elimination of two redundant measurements makes the system unobservable.

d. Critical *k*-tuple: contain *k* non-critical measurements whose simultaneous elimination from the measurement set makes the system unobservable.

Detection and identification of bad data in a system depend on the configuration of meters in that system. If a bad data appears in critical measurements, then it cannot be detected using any method. In other words, a single measurement containing bad data can be identified if and only if it is not critical and it does not belong to a critical pair.

2.2.3 Leverage Points

An outlier can be either a bad data in one of the measurements z or in the factor space of regression which is here an n-dimensional space of the Jacobian matrix H rows. When there is an outlier in the factor space, one of the rows of H will lie away from the rest of the factors. The corresponding measurement will have an undue influence on the state estimate and is referred as the leverage point in regression [1].

The residual of a measurement corresponding to a leverage point will be very small even when it contains a large error which behaves like a critical measurement whose residual is exactly zero. However, the elimination of a leverage point does not render the system unobservable. Therefore, identification of a leverage point containing bad data is very difficult unless a robust method such as IRLS is used.

The occurrence of leverage points in power system is generally related to low measurement redundancy. The following conditions are known to create leverage points [1]:

- An injection measurement placed at a bus which is incident to many branches.
- An injection measurement placed at a bus which is incident to branches of very different impedance values.
- Flow measurements along branches whose impedances are very different from those of other branches in the system.
- Using a very large weight for a specific measurement.

Leverage points can be identified utilizing the projection statistics of the measurements; that is, a measurement i is a leverage point if [1]:

$$PS_i > b_i \tag{2.41}$$

Where the parameters of inequality (2.41) are as defined in equations (2.12) and (2.14).

2.3 Inclusion of Current Phasors in a State Estimator

2.3.1 Current Magnitudes in Conventional Estimator

Line current magnitudes are excluded from conventional state estimators although they are widely available in substations; because their use will lead to various numerical and/or observability problems which may preclude the estimation function. Some of these difficulties are [1]:

- For flat start, the Jacobian elements are undefined if the line current magnitudes are used. Hence, any observability analysis cannot be performed.
- Abrupt changes around the origin of the current Jacobian terms may cause convergence issues for lightly loaded lines.
- The possibility of multiple solutions to the estimation problem especially in the absence of power measurements; since the phase angles can have two opposite values $\pm\delta$. This defies the observability notion.

2.3.2 Synchronized Phasor Current Measurements

The use of phasor measurement units will enhance the performance of state estimators. PMUs can provide voltage phasor measurements as well as branch current phasors. A hybrid estimator which consists of both SCADA and PMU measurements, will be adopted in this work when considering PMU measurements.

There are three methods of incorporating current phasor measurements into a state estimator [10]:

1. Current phasor magnitude and phase angle – polar measurement.
2. Real and imaginary parts of the complex current measurement.
3. Pseudo-voltage measurement with the help of current measurement and known line parameters.

These three methods are presented and evaluated in [10] and they found that Method 2 gave best performance in terms of convergence characteristics and estimation accuracy. Hence, this method – presented in detail in [11] – will be adopted in this thesis.

For the typical π-model branch – Figure 2.1 – the branch current expression is given by:

$$I_{ij} = (g_{si} + jb_{si})V_i + \left(V_i - V_j\right)\left(g_{ij} + jb_{ij}\right) = I_{ij,r} + jI_{ij,i} \tag{2.42}$$

$$I_{ij,r} = \left(g_{ij} + g_{si}\right)V_i \cos\delta_i - g_{ij}V_j \cos\delta_j - (b_{ij} + b_{si})V_i sin\delta_i + b_{ij}V_j \sin\delta_j \tag{2.43}$$

$$I_{ij,i} = \left(g_{ij} + g_{si}\right)V_i \sin\delta_i - g_{ij}V_j \sin\delta_j + (b_{ij} + b_{si})V_i \cos\delta_i - b_{ij}V_j \cos\delta_j \tag{2.44}$$

Where $I_{ij,r}$ and $I_{ij,i}$ denote the real and imaginary parts of the branch current from node i to j.

The corresponding Jacobian sub-matrix can be expressed as:

$$H_I = \begin{bmatrix} \frac{\partial I_{ij,r}}{\partial \delta} & \frac{\partial I_{ij,r}}{\partial V} \\ \frac{\partial I_{ij,i}}{\partial \delta} & \frac{\partial I_{ij,i}}{\partial V} \end{bmatrix} \tag{2.45}$$

The elements of Jacobian sub-matrix $\boldsymbol{H_I}$ are given by:

$$\frac{dI_{ij,r}}{d\delta_i} = -V_i[\sin\delta_i(g_{si} + g_{ij}) + \cos\delta_i(b_{si} + b_{ij})] \tag{2.46}$$

$$\frac{dI_{ij,r}}{d\delta_j} = V_j(b_{ij}\cos\delta_j + g_{ij}\sin\delta_j) \tag{2.47}$$

$$\frac{dI_{ij,r}}{dV_i} = cos\delta_i(g_{si} + g_{ij}) - \sin\delta_i(b_{si} + b_{ij}) \tag{2.48}$$

$$\frac{dI_{ij,r}}{dV_j} = b_{ij}\sin\delta_j - g_{ij}\cos\delta_j \tag{2.49}$$

$$\frac{dI_{ij,i}}{d\delta_i} = -V_i[\sin\delta_i(b_{si} + b_{ij}) - \cos\delta_i(g_{si} + g_{ij})] \tag{2.50}$$

$$\frac{dI_{ij,i}}{d\delta_j} = V_j(b_{ij}\sin\delta_j - g_{ij}\cos\delta_j) \tag{2.51}$$

$$\frac{dI_{ij,i}}{dV_i} = sin\delta_i(g_{si} + g_{ij}) + \cos\delta_i(b_{si} + b_{ij}) \tag{2.52}$$

$$\frac{dI_{ij,i}}{dV_j} = -b_{ij}\cos\delta_j - g_{ij}\sin\delta_j \tag{2.53}$$

The precision of phasor data is evaluated in polar form [12]. The branch current phasors in rectangular form are regarded as indirect measurements in control centers. Therefore, the error covariance matrix of indirect measurements is derived according to the known error variances of the direct – polar – measurements [11]. The error variances due to current measurement transformation can be obtained by:

$$\sigma_{I_{ij,r}}^2 = \left(\frac{dI_{ij,r}}{dI_{ij}}\right)^2 \sigma_{I_{ij}}^2 + \left(\frac{dI_{ij,r}}{d\theta_{I_{ij}}}\right)^2 \sigma_{\theta_{I_{ij}}}^2 = (\cos\theta_{I_{ij}})^2 \sigma_{I_{ij}}^2 + (I_{ij}\sin\theta_{I_{ij}})^2 \sigma_{\theta_{I_{ij}}}^2 \qquad (2.54)$$

$$\sigma_{I_{ij,i}}^2 = \left(\frac{dI_{ij,i}}{dI_{ij}}\right)^2 \sigma_{I_{ij}}^2 + \left(\frac{dI_{ij,i}}{d\theta_{I_{ij}}}\right)^2 \sigma_{\theta_{I_{ij}}}^2 = (\sin\theta_{I_{ij}})^2 \sigma_{I_{ij}}^2 + (I_{ij}\cos\theta_{I_{ij}})^2 \sigma_{\theta_{I_{ij}}}^2 \qquad (2.55)$$

Where $\sigma_{I_{ij,r}}^2$ and $\sigma_{I_{ij,i}}^2$ are the error variances of $I_{ij,r}$ and $I_{ij,i}$ respectively.

These diagonal elements can be easily combined with conventional covariance matrix $\boldsymbol{R}$ – equation 2.2 – to form the hybrid error covariance matrix which should be updated each iteration [11].

CHAPTER 3

OPTMAL PLACEMENT OF PHASOR MEASUREMENT UNITS

3.1 Overview of Phasor Measurement Units

A disadvantage of measurements collected by SCADA is that they do not include the phase angle of bus voltages nor line currents. In addition, these measurements are not synchronized which makes the state estimation results imprecise and even inaccurate in the presence of bad errors. Synchro-phasors or real-time voltage as well as current phasors are provided by (PMU) phasor measurement units. They are digital devices with a very high sampling rate and accuracy, synchronized by receiving a common timing signal from global positioning satellite (GPS) clock [13]. Compared with conventional SCADA data, PMU measurements achieve synchronization precision of one microsecond and magnitude accuracy of 0.1% [14].

When a PMU is placed at a bus, it can provide synchronized measurements of voltage phasor of that bus as well as current phasors of some or all lines connected to that bus. These measurements are linear functions of the states and can be solved directly without iterations. Voltage and current phasor measurements can be included in the measurement vector z and the conventional estimator can be extended to process these measurements as presented in [13].

PMUs can be used in various applications of power system such as measuring magnitude and frequency of phasors, adaptive relaying, instability prediction, improved control, and state estimation [15]. Over the years, more numbers of PMUs are needed to improve monitoring, protection, and control of power system. Because of the high price of PMUs and the cost of communication services connected with them, it is not possible to place a unit at each bus of the network. Rather, a certain number of PMUs are required to be placed at certain locations which make the system fully observable. This is known as the optimal placement of PMUs problem and there are numerous publications in this field. References [16 – 19] give an extensive review of the problem in the literature. Other than full observability goal, there are secondary objectives of PMUs placement which include topology errors detection, measurement error detection, and parameter error detection and identification [14]. However, the interest of this research is in the work related to the use of PMUs for conventional static state estimation formulation and solution accuracy. Multi-area and dynamic state estimations are beyond the scope of this thesis.

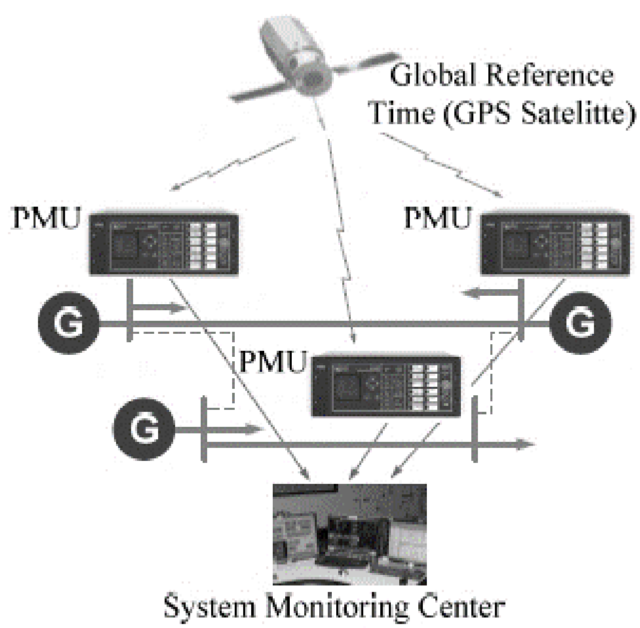

Figure 3.1: Wide area measurement system based on PMUs [16].

3.2 Literature Survey

Phadke was the first to apply PMUs for state estimation in [20] where he proposed providing each node with a PMU which measures the phasor voltage of the bus and the phasor currents of all incident branches to this bus. This makes the state estimation a linear problem with constant gain matrix. The optimal placement problem of PMUs to minimize the number of unobservable buses was then considered in [8] where the concept of spanning measurement subgraph was introduced. The problem was solved through a dual search technique which used both a simulated annealing-based method and a modified bisecting search. [8] found that one fourth to one third of the buses had to be equipped with PMUs to make the system observable. In [21], system observability by minimizing the installation cost – and hence, the number – of PMUs problem is formulated as a binary integer programming and is extended in [22] to account for the loss of single PMU to investigate the effect of PMU failure on state estimation. In addition to PMU measurements, the problem formulation of [21, 22] can consider conventional measurements of injections and flows if they exist in the system. It was found that conventional measurements reduce the required number of PMUs which makes the system observable. The authors in [23] propose a method for complete system numerical observability with single branch outage and single measurement loss. The selection of candidate solutions is based on the lowest condition number of the normalized measurement matrix as a criterion. The binary integer programming is then applied to select the optimal redundant measurements as well as the minimal number of PMU locations. Paper [24] shows that the solution in [23] is not truly optimal. The approach proposed in [24] ensures observability for single PMU outage and gives global optimal number and location of PMUs considering single branch outage. The exhaustive binary search method proposed overcomes the limitations of integer programming and the uncertainties of evolutionary algorithms. The method is examined on standard test systems without considering conventional measurements to provide benchmark solutions to researchers in this field. In [25], integer-based artificial bee colony (ABC) technique is used to find the optimal PMUs numbers and locations for complete network observability. Artificial intelligence techniques such as ABC guarantees a near global solution with properly chosen parameters. This optimization method of [25] is tested on four IEEE systems and it is found that for a given system, strategically placing PMUs at one-third of the buses makes the system entirely observable. The same problem is addressed in paper [26] where a hybrid optimization that combines cuckoo search and genetic algorithm (CS-GA) is used. The presented technique is efficient under normal operating conditions and for single PMU loss, also. Zero injection buses are not considered, and the paper provides benchmark solution for the IEEE 14-, 24-, 30-, and 39- bus test systems.

So far, conventional measurements were excluded from the formulation and only PMU measurements are considered. Based on [22], paper [27] proposes a unified approach for determining the optimal locations and number of PMUs that make the power system observable and therefore can be used for state estimation. The problem is addressed as a binary integer linear programming as in [22] but does not alter the network topology for the inclusion of conventional measurements. Single or multiple PMU loss is considered in the problem. The suggested method is simulated on different IEEE bus systems and the results are said to agree with those published in the literature. A hybrid state estimator model joining the measurements from the conventional WLS state estimator with linear

PMU state estimator is presented in [28]. The model of [28] is linear in rectangular variables. Hence, the solution is non-iterative since the PMU measurements are not directly integrated with traditional inputs. The model is applied on the New England 39-bus test system and it was shown that the addition of few PMUs can remarkably raise the accuracy of the flow and state estimates. The paper does not address the optimal PMUs placement problem.

State estimation using only PMU measurements is not practical in the current time due to cost considerations and needs years to come. A more realistic approach is to use conventional measurements along with PMU measurements to enhance the performance and accuracy of state estimation as presented in [27], [28], [29] and later in [30] with two techniques for combining these measurements with supporting theory and simulations. This approach is considered in some publications where the optimal PMU placement problem objective is to improve the state estimation solution not just the network observability as in [26 – 29]. In [31], an incremental PMU placement approach is applied to reduce the uncertainties of state estimation based on conventional SCADA measurements. This incremental algorithm is adopted from [32] where the covariance of the state-error vector – or the diagonal elements of the gain matrix inverse in WLS method – is used to assess the accuracy of the estimation. Only the voltage phasor of the PMU is considered in [31] and the current phasor is excluded. The authors of [33] formulate the PMU placement problem as an optimal experiment design with a class of well-known optimality criteria in statistics. A greedy algorithm for PMU placement is demonstrated to reduce the state estimation error which is represented as the covariance matrix of the estimated state vector. Only the measured voltage angles of PMUs are considered which might not achieve accurate estimation results as noted by [34]. Paper [34] proposes a heuristic PMU placement approach to enhance the state estimation precision which is assessed by the performance indicator of average Mean Average Percentage Error ($MAPE_{avg}$). In this algorithm, both voltage and current phasors of PMUs are considered and the WLS method is used to calculate state estimation results. The two techniques of incorporating PMU measurements into the estimator [30] are investigated and they achieved similar results.

The optimal placement of PMUs problem is still a hot research topic and appreciable amount of publications are made to attain system observability with smallest number of PMUs to decrease the cost of wide area management system (WAMS). In [35], the cost of phasor data concentrators (PDCs) and their associated communication interface (CI) are considered along with PMUs to be minimized. A new optimization method combining binary imperialistic competition algorithm (BICA) and the Dijkstra algorithm is proposed to find the optimal placement of PMUs and PDC with minimal communication paths and full observability. Paper [36] presents a unified algorithm combining observability and bad data detection as placement objective of PMUs. The algorithm is performed using an improved integer linear programming (ILP) model and is applied on IEEE 14- and 118-bus systems. The authors of [37] applies differential evolution (DE) algorithm for the problem. The formulated optimization problem considers the placement of new PMUs as well as relocating or removing existing ones to cope with continuous changes in the topology of the system. The proposed technique is tested on IEEE 14-, 30-, 39-, and 57-bus systems and the obtained results agree with the published results.

3.3 Modeling of PMUs in a State Estimator

There are two main approaches reported in the literature for including PMUs in the state estimator. The first approach is to process both SCADA and PMU measurements at the same time in a non-linear model. This is known as a one-phase state estimator or as a hybrid estimator and it was covered in the previous chapter – section 2.3. The second approach is to process SCADA measurements using the conventional estimator as a first stage. In the second stage, the PMU measurements along with first stage solutions will be processed in a linear model. This approach is known as a two-phase state estimator. Reference [38] presents a comparative analysis of these two approaches. It shows that the one-phase estimator presents the best accuracy in all test cases since this approach allows the PMU measurements to spread their benefits to all the buses as opposed to the two-phase estimator which enhances only the estimates of the buses monitored by PMUs or adjacent to PMU-monitored buses. This enhancement in the one-stage estimator is at the expense of significant modification in the existing state estimator.

Most of the publications, which deal with the use of PMUs to enhance the performance of state estimator, adopt the traditional WLS estimator. The inclusion of PMU measurements in a robust estimator is rarely discussed in the literature. Paper [39] incorporates PMUs in a weighted least absolute value (WLAV)-based estimator to improve state tracking accuracy and speed. The prime objective of this incorporation is to process the synchronized measurements at their refresh rates. In [40], a two-stage GM-estimator using both SCADA and PMU measurements is proposed to handle bad data measurements. The paper investigates non-Gaussian heavy tailed noise distributions in contrast to the traditionally-assumed Gaussian model. Reference [41] introduces a robust hybrid state estimation which utilizes space and time correlation in adjacent PMUs to reject outliers in buffers and topology errors. The robustness of this estimator is introduced by replacing the WLS with the least trimmed squares (LTS) estimator.

In this thesis, the one-stage state estimator for both SCADA and PMU measurements model is adopted for both WLS and IRLS estimators. The mathematical equations and detailed model is discussed earlier in Section 2.3.

The PMUs are incorporated in the state estimation algorithm under the following assumptions [28]:

1. The phasor measurements and the traditional measurements are taken at the same snapshot – there is no time skew between them.
2. A PMU is always present at the slack bus so that the reference angle of both measurements is the same.
3. When a PMU is installed at a certain bus, it can read the bus voltage phasor as well as all the branch current phasors connected to that bus and flowing away from that bus.

3.4 Heuristic PMU Placement Algorithm

As presented in Section 3.2, most of the publications consider the observability as the main objective of PMUs placement. In this work, however, the algorithm presented in reference [34] will be adopted since it considers enhancing the estimation accuracy as the principal purpose of deploying PMUs in a power system.

Assuming p PMUs need to be placed in a power system of n buses ($p < n$). Each bus can only have one PMU. After each placement of PMUs, state estimation algorithm is implemented, and the estimation accuracy is evaluated by the following indicator:

$$\frac{1}{2n}\sum_{i=1}^{2n}|actual_i - estimated_i| \tag{3.1}$$

The performance indicator is defined as the sum of the absolute difference between base and estimated states. Here, *2n* is the number of states.

This algorithm assumes that the system is already observable by conventional measurements and PMUs are placed to improve the estimation accuracy. Slack bus – usually bus 1 – in the system is considered the reference bus where a PMU is installed to give an accurate reference for other buses. This PMU is not counted in the placement problem.

To place p PMUs in the rest $(n – 1)$ buses in the system, the total number of placement possibilities can be found to be:

$$A_{n-1}^{p} = (n-1)(n-2)(n-3)\cdots(n-p) = \frac{(n-1)!}{(n-p-1)!} \tag{3.2}$$

The proposed heuristic technique will reduce this number of possibilities considerably. Shortly, one can see that the number of placement possibilities will be:

$$A_{n-1}^{p} = (n-1)+(n-2)+(n-3)+\cdots+(n-p) = \frac{n(n-1)-p(p-1)}{2} \tag{3.3}$$

Before describing the algorithm procedure, the variables used are defined as follows:

- z: the measurement data vector with white noise.

- *R*: the measurement error variance vector.
- *p*: number of PMUs needed to be placed in the system.
- *AllBus*: the vector that contains all the candidate bus numbers.
- *BusWithPMU*: the vector that contains the bus numbers which has been placed with PMUs.
- *BusToPlace*: the vector that contains the bus numbers that will be placed with PMUs. It is obtained by excluding *BusWithPMU* from *AllBus*.
- *Num*: the length of vector *BusToPlace*.
- *BusNum*: the variable that contains the bus number that will be placed with a PMU.
- *KnownPMUBus*: the vector that is formed by combining vector *BusWithPMU* and variable *BusNum*.
- *Ind_array*: the vector that contains indicator values of state estimation.

The detailed algorithm procedure is:

1) Choose a conventional meter placement so that the original system is fully observable with known topology – Ybus matrix is known.
2) Read the number of PMUs *p* and vector *AllBus* that contains all the candidate bus numbers.
3) Initialize the vector *BusWithPMU* with a single element that is bus no. 1 – the slack.
4) Set iteration count *i* = 1.
5) Obtain the vector *BusToPlace* that stores the bus numbers that will be placed with PMUs by excluding the elements of vector *BusWithPMU* from vector *AllBus.*
6) Find the length of vector *BusToPlace* and assign it to variable *Num*. Initialize a vector *Ind_array* of length *Num* to store indicator value of state estimation.

7) Set index count $k = 1$ – inner loop.

8) Read the kth value of vector *BusToPlace* and assign it to variable *BusNum*, and obtain vector *KnownPMUBus* by combining the elements of vector *BusWithPMU* and *BusNum*.

9) Modify the initial measurement data vector z and measurement error variance vector R by adding voltage magnitudes and angles as well as current phasor measurements corresponding to buses from vector *KnownPMUBus*.

10) Implement state estimation to obtain indicator value, and store the value in the kth element of the vector *Ind_array*.

11) If $k \leq Num$, increment the index count by 1 and go to step 8; otherwise go to step 12.

12) Find the bus number with the minimum indicator value from vector *Ind_array*, and add the bus number to vector *BusWithPMU*.

13) If $i \leq p$, increment the iteration count i by 1 and go to step 5; otherwise go to step 14.

14) Print results and stop.

This algorithm provides a very efficient placement technique. It solves the placement problem by providing both the number and bus location of PMUs needed to reach a desired estimation accuracy. It is suitable for any system configuration without the need to adjust the code. It can also be applied to a certain area of the system rather than the whole system. This can be simply achieved by selecting the required buses in vector *AllBus*.

The flowchart of this algorithm is displayed in Figure 3.2 below.

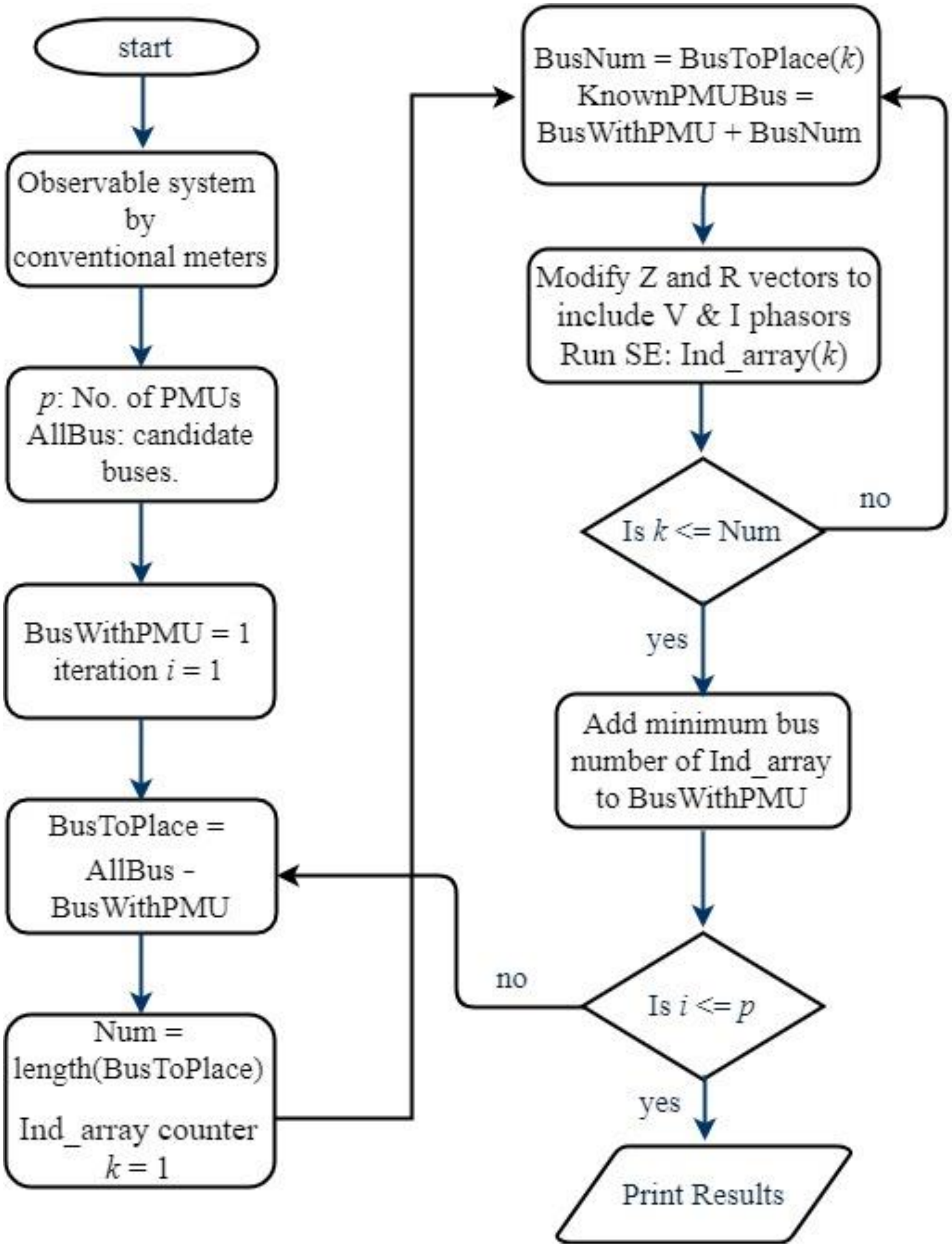

Figure 3.2: Flowchart of OPP algorithm.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Simulation and Data Setup

MATPOWER [42] which is a package of MATLAB files for solving power system operations such as power flow and state estimation is used as a platform for our work since it is purposed as a simulation tool for educators and researchers that is easy to use and modify.

The true or actual values of system states as well as different injections and flows are obtained from power flow results. Random normal "white" noise is generated and added to actual values to simulate measurement meters. The performance indicator is defined as the average of the sum of the absolute difference between actual and estimated states, as given in (4.1). The voltage magnitudes are measured in per unit and the voltage angles are measured in radians.

$$\frac{1}{2n}\sum_{i=1}^{2n}|actual_i - estimated_i| \tag{4.1}$$

This indicator is more representative than the sum of squared residuals especially if one wants to compare various estimators which use indicators other than sum of squared errors such as weighted least absolute value (WLAV) or least measurement rejection (LMR). Dividing the indicator by the number of states makes it suitable to compare several systems with different sizes.

The measurement set is generated by perturbing the base values with random noise using equation (4.2).

$$z_i = actual_i[1 + (-1 + 2 \times RND) \times \sigma_i] \tag{4.2}$$

Where RND: is a random uniform number between 0 and 1. This formula will ensure that the generated measurement will have the same sign as the corresponding actual value. This is important as sign reversal is considered a bad data. The presumed values of standard deviations σ of different types of measurements are listed in Table 4.1. These values are kept fixed for all test cases.

Table 4.1: Standard deviation for different measurement types [43]:

	Measurement Type	**Standard Deviation σ (per unit)**
1	SCADA Voltage magnitude	0.01
2	Real power injection	0.02
3	Reactive power injection	0.04
4	Real power flow	0.02
5	Reactive power flow	0.04
6	PMU Voltage magnitude	0.0001
7	PMU Voltage angle	0.006
8	PMU Current magnitude	0.0001
9	PMU Current angle	0.006

Zero-measurements are assigned large weights since they are considered perfect measurements. Therefore, the standard deviation of a zero-value measurement is divided by 100 to increase its weight.

A redundancy "η" of around two is adopted for various cases since it is more economic and reflects a realistic practice of power utilities.

Bad data measurements are simulated by negating the measurement if it is a power flow or a power injection. Bad voltage measurement is simulated by contaminating the voltage measurement with a value greater/lower than three standard deviations; that is:

$$V_i^{bad} = V_i^{meas} \pm b\sigma_i, \qquad b > 3 \tag{4.3}$$

When a PMU is installed at a certain bus, it reads the bus-voltage phasor and all the branch-current phasors flowing from this bus. Inclusion of a PMU in simulations is done by adding four vectors of meter measurements to account for voltage and current phasors, which are: 1) PMU voltage magnitudes, 2) Voltage angles, 3) Real-part of branch currents, and 4) Imaginary-part of branch currents. The measured values of these metes are set equal to their corresponding true values generated by power flow results without disturbing them by white noise. The corresponding standard deviations of these meters are taken from Table 4.1 for voltage phasors and by applying equations 2.54 and 2.55 for current phasors – as explained in Chapter 2.

The remaining sections present the results of the two state estimation algorithms: WLS and IRLS. A comparison of these two estimators is demonstrated. Finally, results of the optimal placement technique are given for three standard test systems: IEEE 14-, 30-, and 118-bus systems.

4.2 State Estimation Results

The following different bad-data types are applied to the test systems to evaluate the performance of the two estimators – WLS & IRLS:

1. Case 1: no bad data.

2. Case 2: single power flow bad data.
3. Case 3: single power injection bad data.
4. Case 4: single voltage magnitude bad data.
5. Case 5: multiple non-interacting bad data.
6. Case 6: multiple interacting bad data.

Some of the bad data will be applied on leverage points. These cases will be applied twice: first with conventional measurements only, and secondly, with both conventional and PMU measurements.

4.2.1 SE of IEEE 14-Bus System

The SCADA measurement placement of the system is presented in Table 4.2 below.

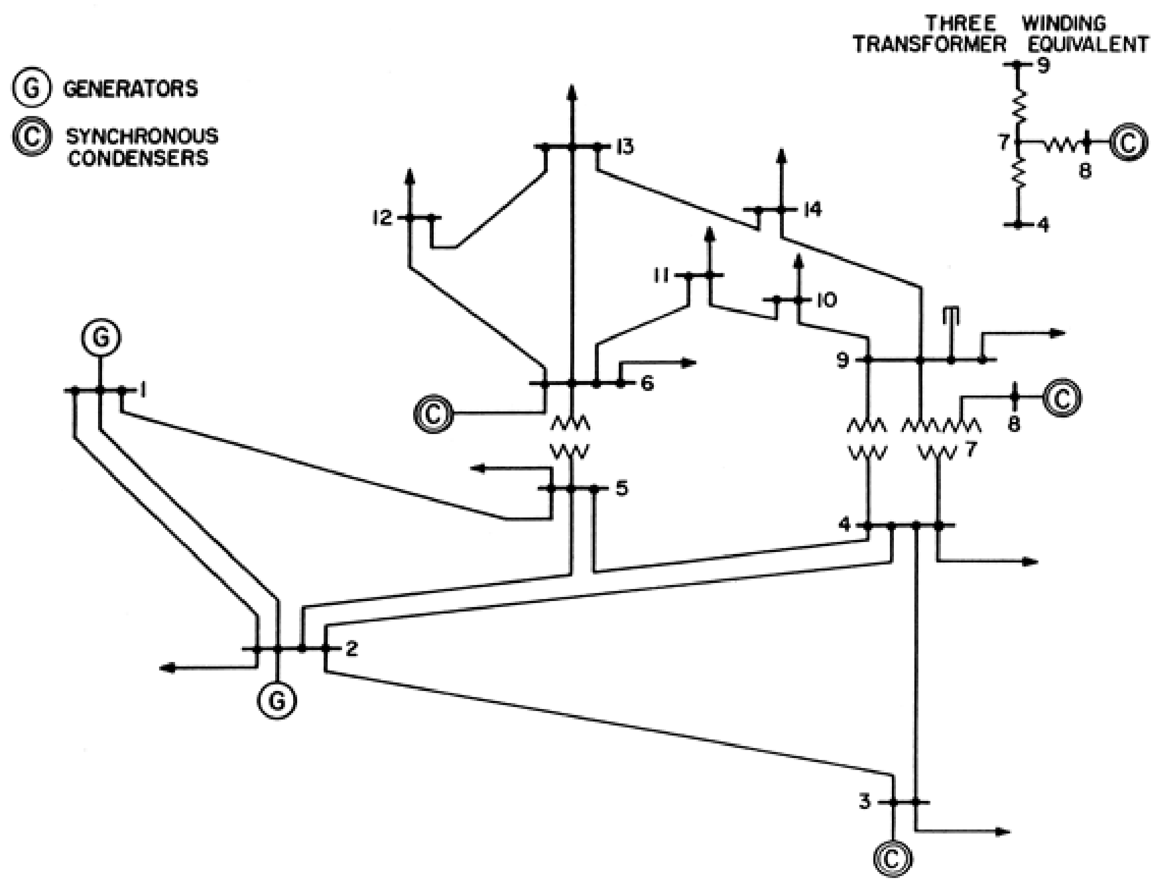

Figure 4.1: IEEE 14-bus test system [44].

Table 4.2: Configuration of SCADA measurement in IEEE 14-bus system:

Redundancy η	2.00
Voltage magnitude buses	1, 3, 11, 13
Power injection buses (real & reactive)	1, 2, 3, 6, 9, 10, 12, 13
Power flow branches (real & reactive)	1-2, 1-5, 2-3, 3-4, 4-7, 6-11, 6-13, 7-8, 9-14, 12-13, 13-14 2-1, 3-2, 5-4, 11-6, 8-7, 13-12

The performance indicator for both estimators for different conventional cases is presented in Table 4.3.

Table 4.3: Performance indicators of IEEE 14-bus system in the presence of different bad data types without PMUs:

Case	Bad Data Locations	Bad Leverage Points	WLS Indicator	IRLS Indicator
1	N/A	–	1.13E-03	1.13E-03
2	P2-3	–	5.16E-03	1.10E-03
3	Q2	Q2	8.08E-03	1.62E-03
4	Vm13	–	7.90E-03	4.21E-03
5	Vm13, Q3, P1-5	Q3	2.33E-02	5.92E-03
6	Vm3, Q2, Q3-2	Q2, Q3-2	1.05E-02	4.93E-03

From the results of Table 4.3, it can be observed that IRLS outperforms WLS since it is able to suppress bad data and give better estimations. With no bad data, the WLS estimator works fine. However, it breaks down with the presence of bad data.

Next, two PMUs are installed at buses 1 and – randomly – 4 which will increase the redundancy to η = 2.52. Table 4.4 presents the performance indicator of the system for the same estimators and with the same bad data values and locations.

Table 4.4: Performance indicators of IEEE 14-bus system in the presence of different bad data types with PMUs:

Case	Bad Data Locations	Bad Leverage Points	WLS Indicator	IRLS Indicator
1	N/A	–	3.61E-04	3.61E-04
2	P2-3	–	1.62E-03	3.48E-04
3	Q2	Q2	8.55E-04	3.60E-04
4	Vm13	–	2.75E-03	1.16E-03
5	Vm13, Q3, P1-5	Q3	2.93E-03	1.31E-03
6	Vm3, Q2, Q3-2	Q2, Q3-2	7.02E-04	4.18E-04

Comparing Tables 4.3 and 4.4, the indicator values after the inclusion of PMUs are lower than the values obtained without placing PMUs which proves the benefits of installing PMUs in a power system. Figure 4.2 lays out these values in a bar chart.

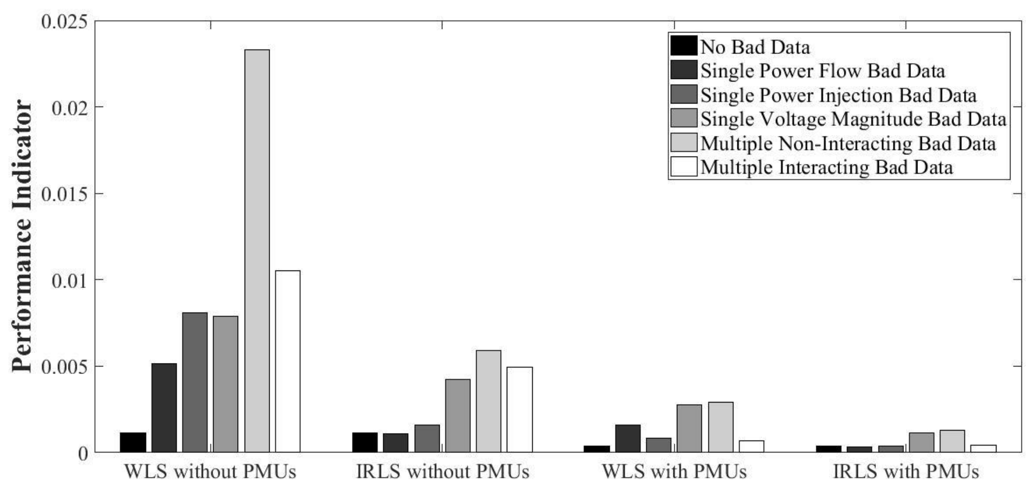

Figure 4.2: Performance indicator of SE in IEEE 14-bus system.

Considering the results of Table 4.4, it is shown that IRLS performs better than WLS whether PMUs are included or not.

4.2.2 SE of IEEE 30-Bus System

The SCADA measurement placement of the system is presented in Table 4.5 below.

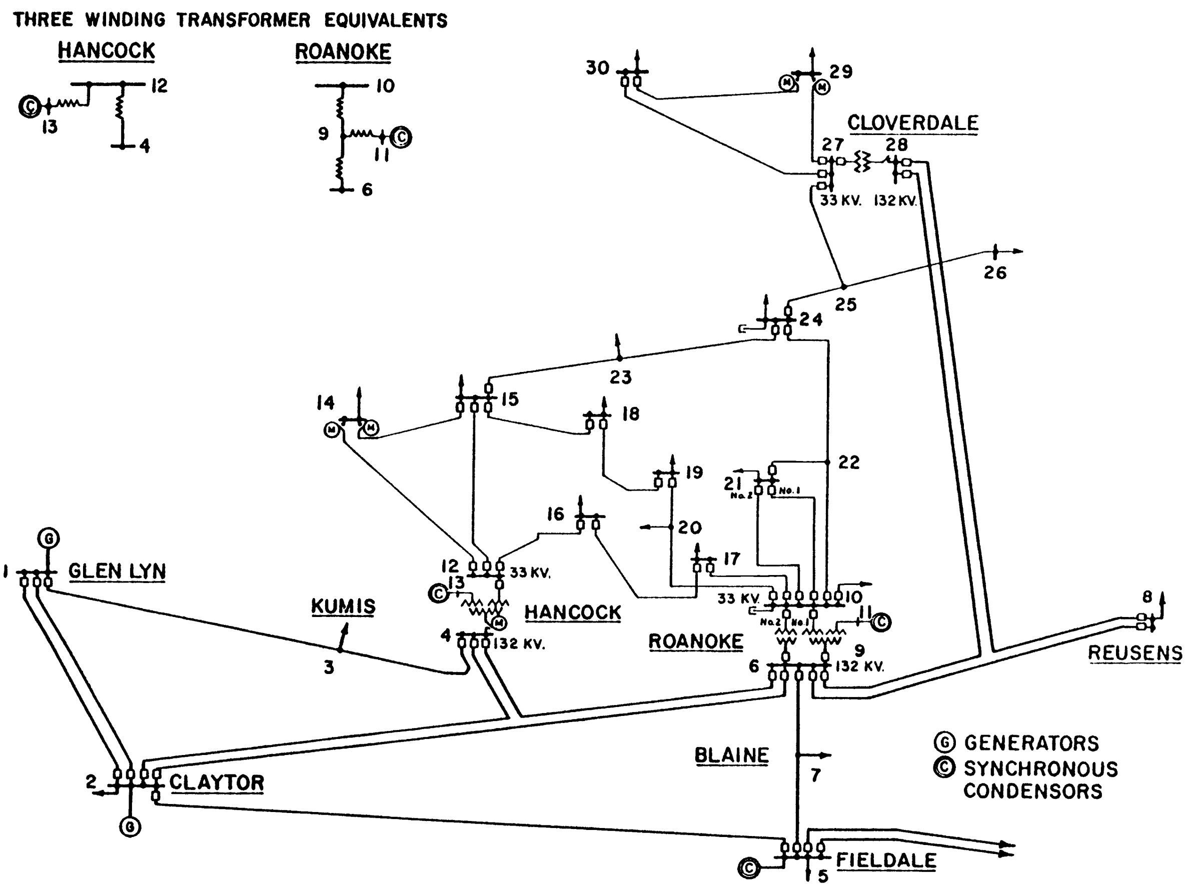

Figure 4.3: IEEE 30-bus test system [44].

Table 4.5: Configuration of SCADA measurement in IEEE 30-bus system:

Redundancy η	2.00
Voltage magnitude buses	1, 3, 8, 10, 12, 18, 21, 24, 26, 28
Power injection buses (real & reactive)	1, 2, 4, 5, 7, 9, 10, 14, 15, 16, 18, 19, 21, 24, 30
Power flow branches (real & reactive)	1-3, 2-4, 2-5, 4-6, 5-7, 6-7, 6-8, 6-9, 9-11, 12-13, 12-14, 12-16, 14-15, 15-18, 18-19, 10-17, 22-24, 23-24, 24-25, 25-26, 25-27, 28-27, 27-30, 29-30 2-1, 4-3, 6-2, 10-9, 12-4, 15-12, 20-19, 20-10, 21-10, 22-21, 23-15, 24-23, 27-28, 30-29, 28-8

The performance indicator for both estimators for different conventional cases is presented in Table 4.6.

Table 4.6: Performance indicators of IEEE 30-bus system in the presence of different bad data types without PMUs:

Case	Bad Data Locations	Bad Leverage Points	WLS Indicator	IRLS Indicator
1	N/A	–	7.12E-04	7.25E-04
2	Q4-6	Q4-6	2.42E-03	8.65E-04
3	Q2	–	6.29E-03	2.62E-03
4	Vm8	–	4.16E-03	1.09E-03
5	Vm8, Q2, P1-3, Q12-4	Q12-4	1.61E-02	3.19E-03
6	P7, Q4, P4-6, Q6-7	Q4, Q6-7	1.34E-02	1.53E-03

Again, it is obvious that IRLS outperforms WLS for different bad data types. In the No-Bad-Data case, the WLS indicator is better than that of IRLS. However, the difference is very small and negligible.

Next, four PMUs are placed randomly at buses 1, 3, 12, and 24 which will increase the redundancy to $\eta = 2.41$. Table 4.7 presents the performance indicator of the system for the same estimators and with the same bad data values and locations.

Table 4.7: Performance indicators of IEEE 30-bus system in the presence of different bad data types with PMUs:

Case	Bad Data Locations	Bad Leverage Points	WLS Indicator	IRLS Indicator
1	N/A	–	3.48E-04	3.59E-04
2	Q4-6	Q4-6	7.75E-04	3.97E-04
3	Q2	–	1.40E-03	7.06E-04
4	Vm8	–	4.10E-04	3.94E-04
5	Vm8, Q2, P1-3, Q12-4	Q12-4	1.33E-03	6.42E-04

6	P7, Q4, P4-6, Q6-7	Q4, Q6-7	9.63E-03	7.30E-04

Results of Table 4.7 display the superiority of IRLS over WLS in terms of suppressing bad data. This is very clear in the last two cases with multiple bad data. The last case shows that PMUs cannot handle bad data with WLS estimator unless they are placed near the bad data locations.

Figure 4.4 lays out the indicator values of the 30-bus system state estimators presented in Tables 4.6 and 4.7 in a bar chart.

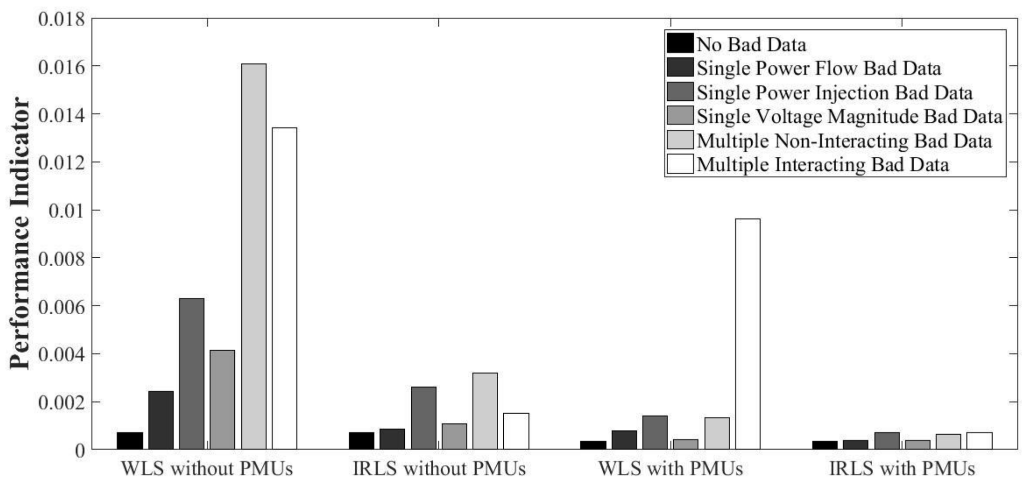

Figure 4.4: Performance indicator of SE in IEEE 30-bus system.

4.2.3 SE of IEEE 118-Bus System

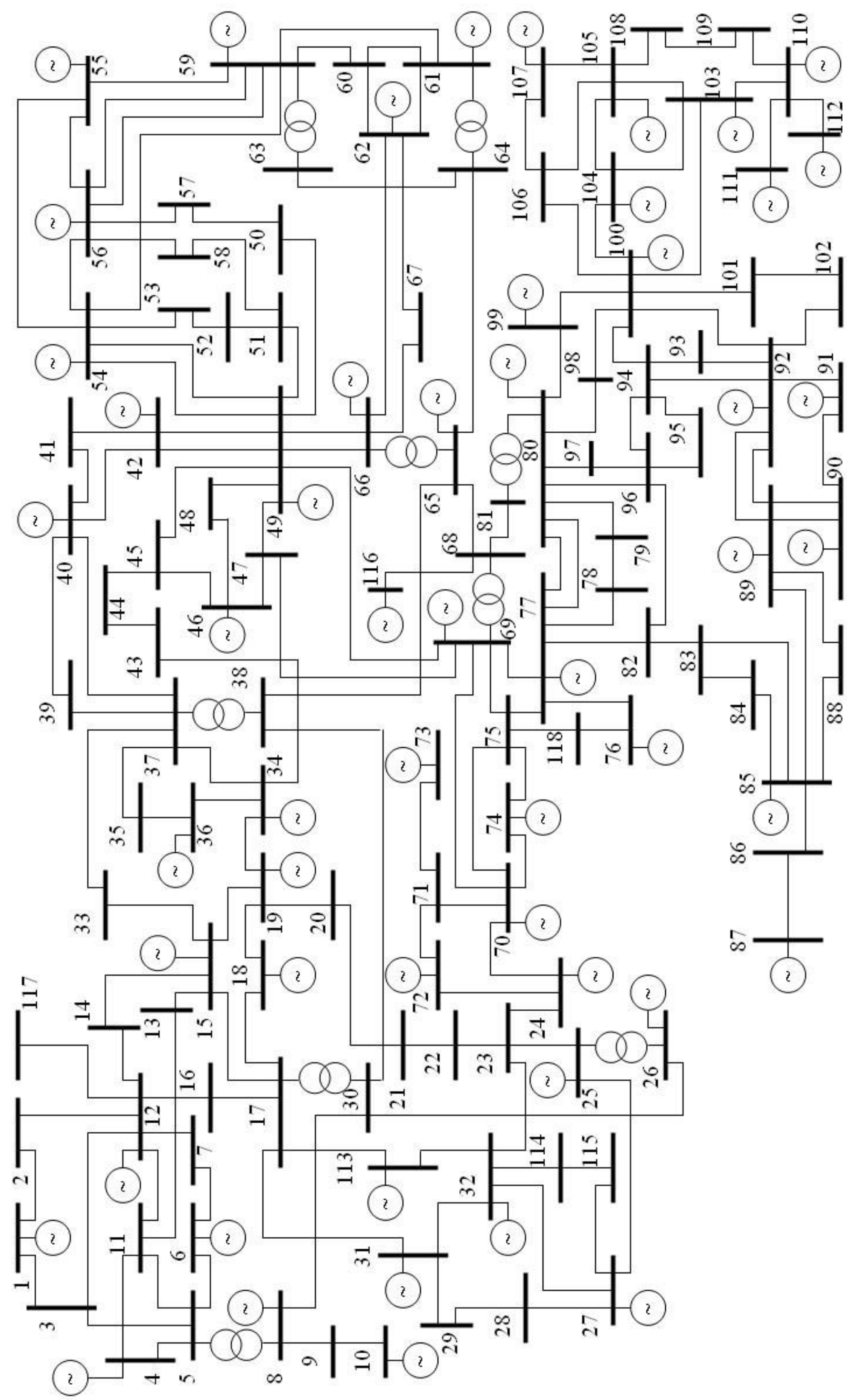

Figure 4.5: IEEE 118-bus test system [45].

The SCADA measurement placement of the system is presented in Table 4.8 below.

Table 4.8: Configuration of SCADA measurement in IEEE 118-bus system:

Redundancy η	1.88
Voltage magnitude buses	2, 3, 4, 5, 9, 12, 15, 17, 18, 21, 23, 24, 25, 27, 28, 29, 30, 34, 36, 37, 40, 42, 44, 45, 46, 49, 51, 53, 54, 56, 57, 59, 62, 63, 64, 68, 69, 70, 71, 73, 75, 76, 77, 80, 82, 85, 86, 91, 92, 100, 101, 102, 103, 105, 107, 110, 111, 112, 113, 114
Power injection buses (real & reactive)	3, 4, 8, 9, 12, 13, 15, 16, 19, 20, 24, 25, 30, 31, 33, 35, 36, 38, 42, 44, 46, 47, 49, 52, 53, 54, 55, 61, 63, 64, 66, 68, 70, 71, 77, 79, 81, 83, 85, 86, 89, 90, 92, 96, 97, 98, 99, 102, 104, 105, 110, 111, 112, 116, 117, 118
Power flow branches (real & reactive)	1-2, 3-5, 5-6, 6-7, 9-10, 4-11, 5-11, 2-12, 7-12, 12-14, 14-15, 17-18, 21-22, 23-24, 28-29, 30-17, 17-31, 23-32, 34-36, 37-40, 39-40, 40-41, 43-44, 34-43, 46-48, 45-49, 52-53, 54-55, 56-57, 50-57, 51-58, 59-60, 60-62, 64-65, 62-67, 65-68, 47-69, 71-72, 71-73, 69-75, 74-75, 76-77, 78-79, 81-80, 77-82, 84-85, 86-87, 85-88, 91-92, 92-93, 94-95, 82-96, 92-100, 95-96, 98-100, 99-100, 100-101, 101-102, 100-106, 105-108, 108-109, 109-110, 17-113, 27-115, 114-115, 12-117, 75-118, 76-118 12-11, 13-11, 15-13, 17-15, 19-18, 21-20, 22-21, 23-22, 25-23, 25-26, 28-27, 30-8, 31-29, 34-19, 37-35, 37-33, 38-30, 42-40, 42-41, 47-46, 49-42, 49-42, 49-48, 50-49, 51-49, 52-51, 54-49, 56-54, 56-55, 58-56, 59-54, 59-56, 61-59, 61-60, 59-63, 64-63, 66-49, 66-62, 66-65, 67-66, 69-49, 69-68, 70-24, 72-24, 74-70, 77-69, 77-75, 80-79, 83-82, 86-85, 87-86, 89-88, 90-89, 94-92, 96-94, 98-80, 100-94, 97-96, 103-100, 105-104, 106-105, 107-106, 109-108, 113-32, 114-32, 116-68

The performance indicator for both estimators for different conventional cases is presented in Table 4.9.

Table 4.9: Performance indicators of IEEE 118-bus system in the presence of different bad data types without PMUs:

Case	Bad Data Locations	Bad Leverage Points	WLS Indicator	IRLS Indicator
1	N/A	–	8.50E-04	9.11E-04
2	P23-32	–	1.03E-02	1.78E-03
3	P66	P66	2.23E-02	9.61E-04
4	Vm36	–	1.25E-03	9.63E-04
5	Vm3, Vm51, P116, Q42, P64-65, Q74-75	P116, P64-65	9.39E-03	1.30E-03
6	Vm18, Vm36, P116, Q19, P116-68, Q19-18	P116, P116-68	7.63E-03	1.28E-03

From the results of Table 4.9 and previous tables, the effectiveness of IRLS algorithm in handling the bad data regardless of the system size.

Next, 16 PMUs are placed randomly at buses 2, 9, 17, 25, 30, 37, 45, 53, 62, 69, 77, 82, 86, 94, 100, and 114 which will increase the redundancy to η = 2.29. Bus 69 is the slack bus of IEEE 118-bus system. Table 4.10 presents the performance indicator of the system for the same estimators and with the same bad data values and locations.

Table 4.10: Performance indicators of IEEE 118-bus system in the presence of different bad data types with PMUs:

Case	Bad Data Locations	Bad Leverage Points	WLS Indicator	IRLS Indicator
1	N/A	–	2.66E-04	2.68E-04
2	P23-32	–	3.13E-03	3.80E-04

3	P66	P66	1.25E-02	3.45E-04
4	Vm36	–	2.92E-04	2.69E-04
5	Vm3, Vm51, P116, Q42, P64-65, Q74-75	P116, P64-65	8.86E-03	3.78E-04
6	Vm18, Vm36, P116, Q19, P116-68, Q19-18	P116, P116-68	4.26E-03	4.41E-04

By comparing the results of WLS estimator with and without PMUs, it is obvious that the inclusion of PMUs does not always enhance the WLS performance – case 5 for example. In the presence of bad data, if the PMUs are not installed near the bad data location, then WLS will not be able to handle the bad data.

The results obtained in this section validate the robustness of IRLS estimator and its outperformance over conventional WLS estimator in the presence of different types of bad data. Figure 4.6 lays out the indicator values of the 118-bus system state estimators presented in Tables 4.9 and 4.10 in a bar chart.

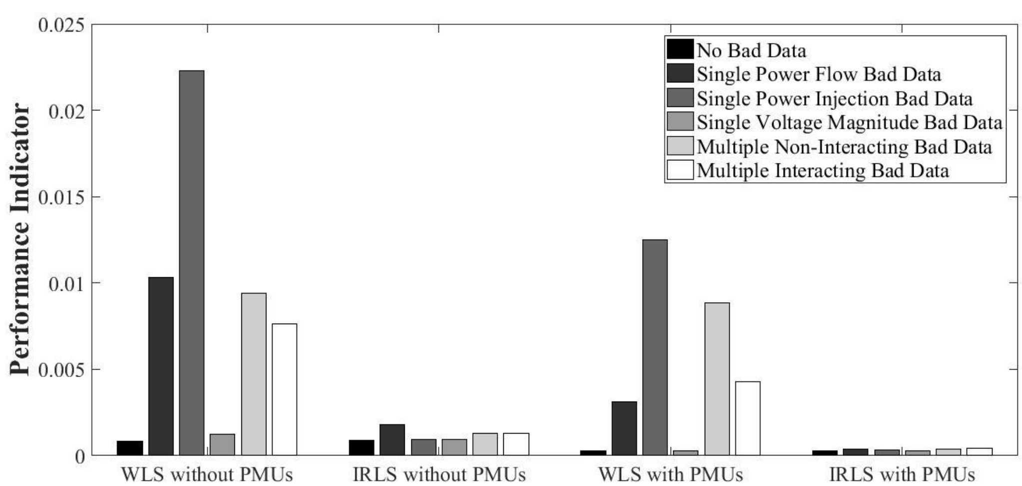

Figure 4.6: Performance indicator of SE in IEEE 118-bus system.

4.3 Optimal PMUs Placement Results

The heuristic placement algorithm presented in section 3.4 will be applied on the three IEEE test systems using the two estimators: WLS and IRLS. To apply the algorithm, the system has to be already observable by conventional measurements. For that reason, the SCADA meter configurations presented in the previous section will be adopted.

4.3.1 OPP in IEEE 14-Bus System

The original system is observable using the meter scheme of Table 4.2. The slack bus – bus 1 – is equipped with a PMU to give accurate reference angle. Figure 4.7 shows the simulation results of this case and their corresponding data are presented in Table 4.11.

Table 4.11: OPP results of IEEE 14-bus system:

No. of PMUs	WLS		IRLS	
	Bus No.	Indicator ($\times10^{-3}$)	Bus No.	Indicator ($\times10^{-3}$)
0	No PMUs	1.1277	No PMUs	1.1277
2	1,6	0.1677	1,6	0.1677
3	1,6,2	0.1065	1,6,2	0.1065
4	1,6,2,7	0.0573	1,6,2,7	0.0573
5	1,6,2,7,13	0.0299	1,6,2,7,13	0.0299
6	1,6,2,7,13,5	0.0248	1,6,2,7,13,5	0.0248
7	1,6,2,7,13,5,9	0.0050	1,6,2,7,13,5,9	0.0050
8	1,6,2,7,13,5,9,3	0.0013	1,6,2,7,13,5,9,3	0.0013
9	1,6,2,7,13,5,9,3,4	0.0007	1,6,2,7,13,5,9,3,4	0.0007
10	1,6,2,7,13,5,9,3,4 ,10	0.0007	1,6,2,7,13,5,9,3,4 ,10	0.0007
11	1,6,2,7,13,5,9,3,4 ,10,14	0.0007	1,6,2,7,13,5,9,3,4 ,10,14	0.0007

12	1,6,2,7,13,5,9,3,4,10,14,11	0.0007	1,6,2,7,13,5,9,3,4,10,14,11	0.0007
13	1,6,2,7,13,5,9,3,4,10,14,11,12	0.0007	1,6,2,7,13,5,9,3,4,10,14,11,12	0.0007
14	1,6,2,7,13,5,9,3,4,10,14,11,12,8	0.0007	1,6,2,7,13,5,9,3,4,10,14,11,12,8	0.0007

The first column in Table 4.11 gives the number of PMUs being placed in the system. The second column gives the bus locations of these PMUs. And the third column lists the performance indicator values.

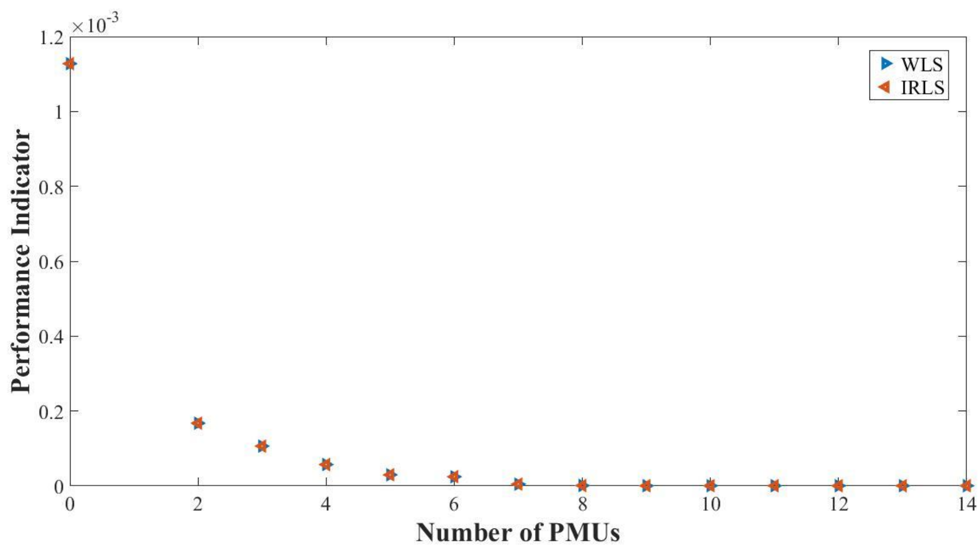

Figure 4.7: OPP results of IEEE 14-bus system.

Generally, as the number of installed PMUs increases, the performance indicator decreases and hence, the estimation accuracy enhances. This continues until reaching the lowest possible indicator. For the current system, it can be seen from Figure 4.7 that the smallest number of PMUs required to enhance the state estimation is four.

Practically, a power utility will not place PMUs in all or even most of the buses. However, the obtained table helps planning engineers to determine the optimal bus locations to install PMUs when they have only a limited number of PMUs.

Comparing the results of the two estimators: WLS and IRLS, it is seen that the indicator values as well as the bus locations are the same. Both estimators prioritize buses with low local redundancy.

To further investigate this optimal placement, the 14-bus system will be tested in the presence of bad data again – retest of Table 4.4 – to compare the effect of locations of PMUs. Table 4.12 shows the results of this comparison. And Figure 4.8 displays these results in a bar chart which is more presentable.

Table 4.12: Comparison between random and optimal placement of 2 PMUs in the 14-bus system:

Case	Bad Data Locations	Random Placement		Optimal Placement	
		WLS	IRLS	WLS	IRLS
1	N/A	3.61E-04	3.61E-04	1.68E-04	1.68E-04
2	P2-3	1.62E-03	3.48E-04	1.98E-03	1.53E-04
3	Q2	8.55E-04	3.60E-04	2.19E-03	1.84E-04
4	Vm13	2.75E-03	1.16E-03	1.89E-04	1.72E-04
5	Vm13, Q3, P1-5	2.93E-03	1.31E-03	4.52E-04	3.69E-04
6	Vm3, Q2, Q3-2	7.02E-04	4.18E-04	1.94E-03	2.11E-04

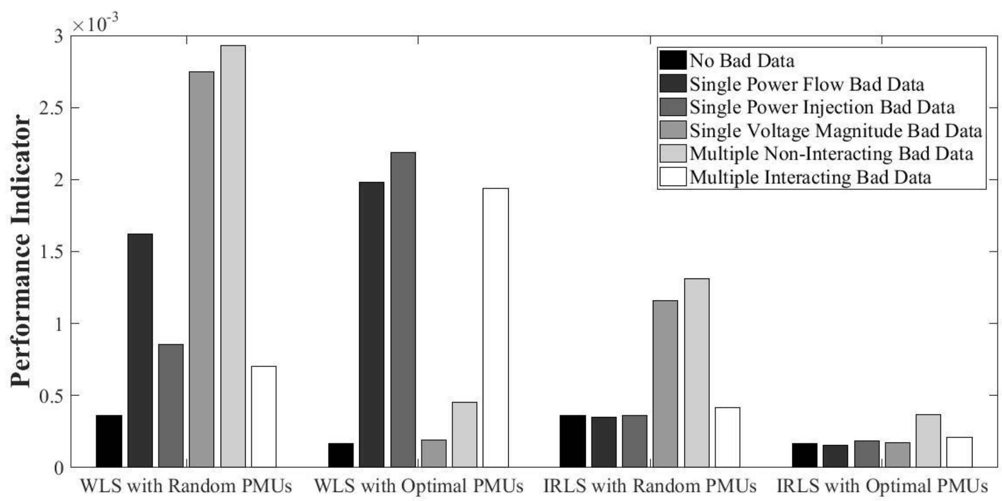

Figure 4.8: Comparison between random and optimal placement of 2 PMUs in the 14-bus system.

With no bad data, the indicators of optimal placement surpass those of random placement as expected. With bad data, however, one can see that the WLS indicator fails at some cases highlighted in the table since PMUs are placed away from the faulted buses now.

4.3.2 OPP in IEEE 30-Bus System

The original system is observable using the meter scheme of Table 4.5. The slack bus – bus 1 – is equipped with a PMU to give accurate reference angle. Figure 4.9 shows the simulation results of this case and their corresponding data are presented in Table 4.13.

Table 4.13: OPP results of IEEE 30-bus system:

No. of PMUs	WLS		IRLS	
	Bus No.	Indicator ($\times 10^{-3}$)	Bus No.	Indicator ($\times 10^{-3}$)
0	No PMUs	0.7119	No PMUs	0.7255
2	6	0.2645	6	0.2645
3	22	0.1888	22	0.1888
4	25	0.1351	25	0.1351
5	12	0.0831	12	0.0828
6	29	0.0514	29	0.0511
7	18	0.0424	18	0.0421
8	24	0.0370	24	0.0367
9	23	0.0348	27	0.0344
10	27	0.0316	23	0.0315
11	9	0.0293	9	0.0292
12	15	0.0278	15	0.0275
13	21	0.0249	21	0.0247
14	19	0.0238	19	0.0235
15	17	0.0232	17	0.0229
16	26	0.0231	26	0.0229
17	13	0.0231	13	0.0228

18	30	0.0231	30	0.0228
19	20	0.0230	20	0.0228
20	14	0.0230	14	0.0228
21	11	0.0230	11	0.0228
22	7	0.0232	7	0.0229
23	16	0.0227	16	0.0225
24	2	0.0187	2	0.0188
25	28	0.0040	28	0.0040
26	5	0.0031	5	0.0032
27	8	0.0030	8	0.0031
28	3	0.0034	3	0.0034
29	10	0.0004	10	0.0004
30	4	0.0006	4	0.0006

In this table, the second column does not include all the buses equipped by PMUs. Since the algorithm places PMUs incrementally bus by bus according to lowest indicator value, only the last bus number to be equipped with a PMU is written in the second column instead of writing the optimal locations every step.

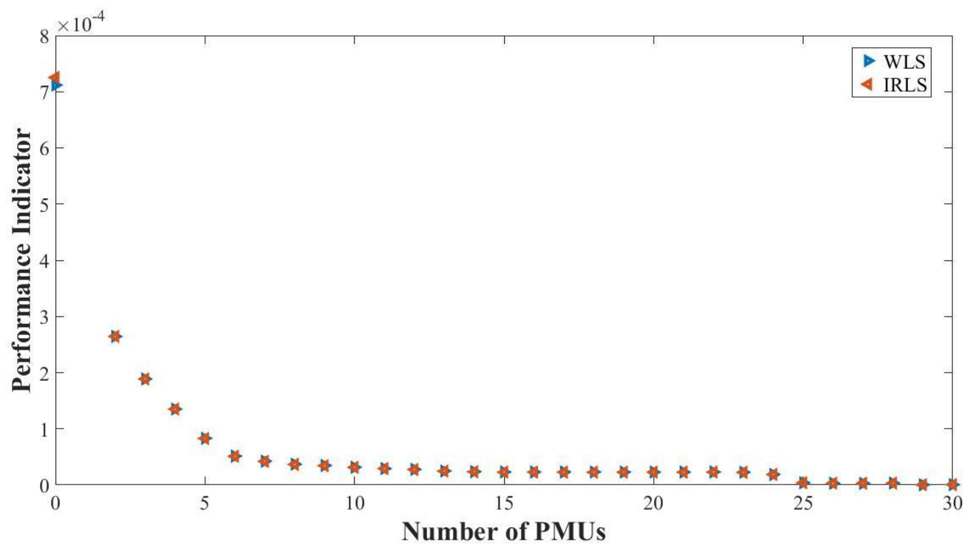

Figure 4.9: OPP results of IEEE 30-bus system.

Like the 14-bus system, the bus locations for PMUs are the same for both estimators. Also, it should be noted that this placement algorithm is dependent on the initial SCADA meters placement. Changing that configuration – or changing the redundancy level – will change the PMU-placement results certainly.

Both estimators provide comparable indicator values – since the data tested contain no bad data – and similar locations for most steps. The smallest number of PMUs required to enhance the estimation is 6 as seen from Figure 4.9.

To further investigate this optimal placement, the 30-bus system will be tested in the presence of bad data again – retest of Table 4.7 – to compare the effect of locations of PMUs. Table 4.14 shows the results of this comparison.

Table 4.14: Comparison between random and optimal placement of 4 PMUs in the 30-bus system:

Case	Bad Data Locations	Random Placement		Optimal Placement	
		WLS	IRLS	WLS	IRLS
1	N/A	3.48E-04	3.59E-04	1.35E-04	1.35E-04
2	Q4-6	7.75E-04	3.97E-04	8.09E-04	1.33E-04
3	Q2	1.40E-03	7.06E-04	8.72E-04	1.50E-04
4	Vm8	4.10E-04	3.94E-04	1.36E-04	1.35E-04
5	Vm8, Q2, P1-3, Q12-4	1.33E-03	6.42E-04	1.88E-03	1.94E-04
6	P7, Q4, P4-6, Q6-7	9.63E-03	7.30E-04	1.50E-02	8.78E-04

Like the 14-bus system, the results of 30-bus system show that the optimal placement of PMUs enhances the indicator values for some cases of bad data but not all because the PMUs are moved away from the faulted buses. Figure 4.10 displays these results in a bar chart which is more presentable.

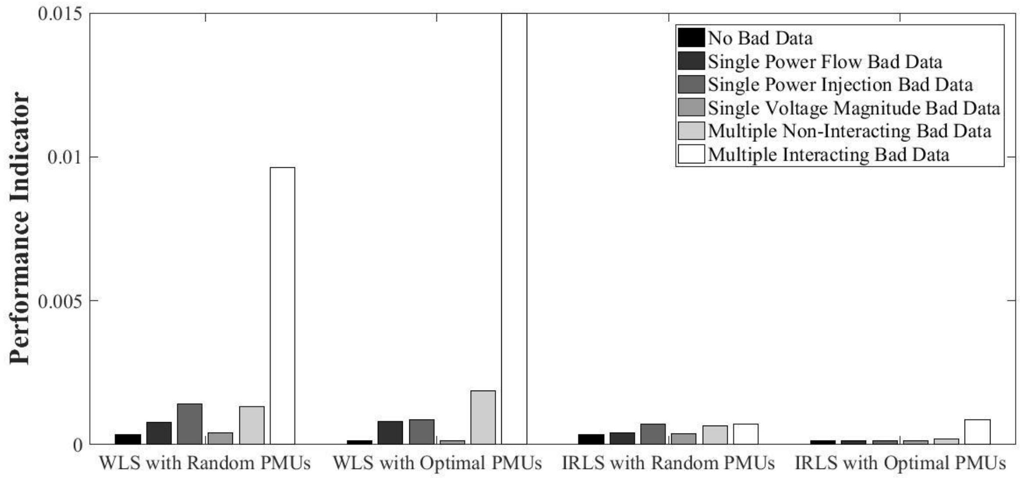

Figure 4.10: Comparison between random and optimal placement of 4 PMUs in the 30-bus system.

4.3.3 OPP in IEEE 118-Bus System

The original system is observable using the meter scheme of Table 4.8. The slack bus – bus 69 – is equipped with a PMU to give accurate reference angle. Figure 4.11 shows the simulation results of this case and their corresponding data are presented in Table 4.15.

Table 4.15: OPP results of IEEE 118-bus system:

No. of PMUs	WLS		IRLS	
	Bus No.	Indicator ($\times 10^{-3}$)	Bus No.	Indicator ($\times 10^{-3}$)
0	No PMUs	0.8500	No PMUs	0.9115
2	100	0.6639	26	0.6754
3	30	0.5702	38	0.5738
4	25	0.4922	100	0.4724
5	64	0.4118	32	0.4177
6	32	0.3608	64	0.3759
7	1	0.3259	16	0.3502
8	49	0.2955	92	0.3270
9	92	0.2723	84	0.3020
10	84	0.2480	9	0.2825

11	21	0.2311	40	0.2627
12	37	0.2154	56	0.2454
13	9	0.1983	21	0.2295
14	45	0.1858	68	0.2174
15	70	0.1748	71	0.1982
16	68	0.1645	34	0.1851
17	34	0.1552	45	0.1763
18	105	0.1475	2	0.1683
19	62	0.1402	28	0.1607
20	23	0.1343	105	0.1536
21	43	0.1244	51	0.1467
22	22	0.1166	86	0.1400
23	31	0.1106	75	0.1332
24	86	0.1049	50	0.1285
25	75	0.0993	37	0.1242
26	110	0.0948	11	0.1199
27	56	0.0907	19	0.1163
28	13	0.0860	27	0.1126
29	48	0.0827	110	0.1094
30	109	0.0794	109	0.1055
31	53	0.0769	44	0.1024
32	12	0.0746	67	0.0998
33	7	0.0710	60	0.0969
34	5	0.0683	18	0.0944
35	80	0.0665	80	0.0925
36	94	0.0638	91	0.0885
37	90	0.0607	82	0.0842
38	83	0.0518	48	0.0824
39	82	0.0420	95	0.0808
40	77	0.0402	53	0.0794

41	59	0.0384	117	0.0781
42	33	0.0373	7	0.0763
43	65	0.0362	1	0.0751
44	18	0.0353	90	0.0741
45	19	0.0342	83	0.0692
46	40	0.0332	94	0.0645
47	38	0.0313	85	0.0585
48	24	0.0304	79	0.0528
49	103	0.0296	63	0.0512
50	76	0.0288	43	0.0494
51	54	0.0280	4	0.0484
52	78	0.0274	33	0.0473
53	27	0.0268	93	0.0464
54	29	0.0261	70	0.0454
55	93	0.0256	77	0.0446
56	51	0.0250	89	0.0428
57	10	0.0247	76	0.0420
58	4	0.0245	103	0.0411
59	108	0.0243	55	0.0406
60	66	0.0241	6	0.0400
61	61	0.0239	59	0.0397
62	35	0.0237	54	0.0391
63	20	0.0236	31	0.0387
64	8	0.0209	108	0.0384
65	26	0.0175	47	0.0383
66	17	0.0166	23	0.0302
67	2	0.0159	17	0.0183
68	44	0.0153	12	0.0111
69	46	0.0150	20	0.0091
70	47	0.0146	8	0.0083

71	3	0.0143	42	0.0080
72	15	0.0141	30	0.0077
73	104	0.0140	22	0.0075
74	87	0.0139	25	0.0073
75	99	0.0139	35	0.0072
76	74	0.0138	15	0.0071
77	95	0.0138	14	0.0069
78	112	0.0137	3	0.0069
79	79	0.0137	74	0.0068
80	42	0.0137	112	0.0068
81	41	0.0136	88	0.0068
82	116	0.0136	29	0.0067
83	39	0.0136	10	0.0067
84	63	0.0136	102	0.0067
85	97	0.0136	52	0.0067
86	107	0.0136	113	0.0067
87	113	0.0136	107	0.0067
88	81	0.0136	58	0.0067
89	73	0.0136	57	0.0067
90	67	0.0136	114	0.0067
91	28	0.0136	111	0.0067
92	111	0.0136	97	0.0067
93	52	0.0136	5	0.0067
94	102	0.0136	61	0.0067
95	98	0.0136	118	0.0067
96	101	0.0135	72	0.0067
97	96	0.0135	115	0.0067
98	55	0.0135	87	0.0067
99	117	0.0135	106	0.0067
100	50	0.0135	73	0.0067

101	60	0.0134	36	0.0067
102	58	0.0134	96	0.0067
103	118	0.0134	81	0.0067
104	115	0.0134	13	0.0067
105	72	0.0134	39	0.0067
106	57	0.0134	99	0.0067
107	36	0.0134	104	0.0066
108	106	0.0134	98	0.0066
109	114	0.0134	46	0.0066
110	91	0.0134	101	0.0067
111	85	0.0099	41	0.0067
112	89	0.0046	78	0.0069
113	88	0.0046	66	0.0072
114	71	0.0048	49	0.0046
115	6	0.0052	65	0.0024
116	11	0.0052	62	0.0023
117	14	0.0052	116	0.0023
118	16	0.0053	24	0.0038

Results of Table 4.15 confirms the validity of the PMUs placement algorithm for any system regardless of its size. Certainly, as the number of system buses increases, the execution time of the placement code increases. But, no utility will place a PMU at each bus of the system. Placing restricted number of buses in an N-bus system will not require too much time for the placing code to be carried out.

Once again, it is shown that both state estimators – WLS and IRLS – provide comparable values of performance indicator for the same number of PMUs. Unlike previous systems, however, the bus locations for PMUs are different for many steps. Also, it is noticed that increasing the number of PMUs will improve the estimation accuracy up to a certain number. Beyond that, the indicator value saturates.

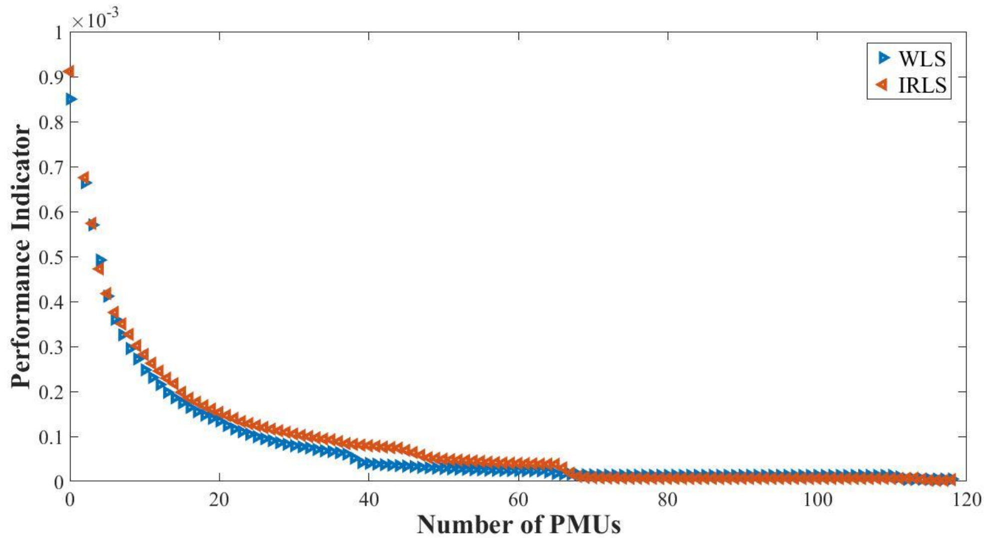

Figure 4.11: OPP results of IEEE 118-bus system.

To further investigate this optimal placement, the 118-bus system will be tested in the presence of bad data again – retest of Table 4.10 – to compare the effect of locations of PMUs. Table 4.16 shows the results of this comparison.

Table 4.16: Comparison between random and optimal placement of 16 PMUs in the 118-bus system:

Case	Bad Data Locations	Random Placement		Optimal Placement	
		WLS	IRLS	WLS	IRLS
1	N/A	2.66E-04	2.68E-04	1.64E-04	1.85E-04
2	P23-32	3.13E-03	3.80E-04	4.11E-03	2.55E-04
3	P66	1.25E-02	3.45E-04	4.04E-03	2.36E-04
4	Vm36	2.92E-04	2.69E-04	1.89E-04	1.85E-04
5	Vm3, Vm51, P116, Q42, P64-65, Q74-75	8.86E-03	3.78E-04	4.58E-04	2.38E-04
6	Vm18, Vm36, P116, Q19, P116-68, Q19-18	4.26E-03	4.41E-04	4.65E-04	3.41E-04

Results of Table 4.16 shows that the optimal placement of PMUs improves the estimation for most cases. Figure 4.12 displays these results in a bar chart which is more presentable.

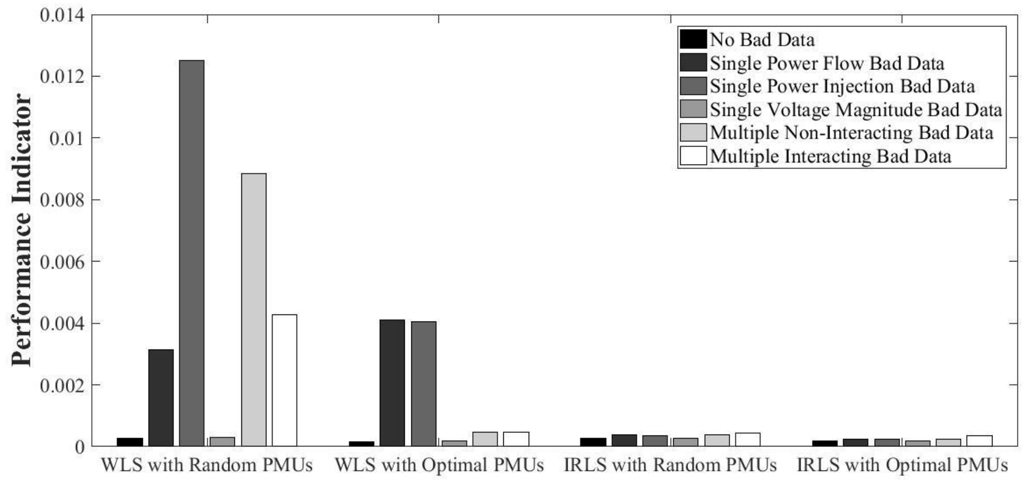

Figure 4.12: Comparison between random and optimal placement of 16 PMUs in the 118-bus system.

4.3.4 Heuristic OPP with Constraints

In this subsection, the optimal placement of PMUs in IEEE 14-bus system will be performed again using the heuristic algorithm but with equipping some buses – other than the slack bus – with PMUs before running the code to explore the robustness of the heuristic technique. Only the WLS estimator will be considered here. Table 4.17 shows the results of two cases: the first case is without constraints which is obtained from Table 4.11, and the second case is obtained by installing PMUs at buses 4 and 6 in addition to the slack bus – bus 1.

Table 4.17: Constrained-Heuristic OPP results of IEEE 14-bus system:

No. of PMUs	**Without Constraints**		**With Constraints**	
	Bus No.	**Indicator ($\times 10^{-3}$)**	**Bus No.**	**Indicator ($\times 10^{-3}$)**
3	1,6,2	0.1065	1,6,4	0.11034

4	1,6,2,7	0.0573	1,6,4,3	0.0617
5	1,6,2,7,13	0.0299	1,6,4,3,9	0.0165
6	1,6,2,7,13,5	0.0248	1,6,4,3,9,7	0.0087
7	1,6,2,7,13,5,9	0.0050	1,6,4,3,9,7,10	0.0018
8	1,6,2,7,13,5,9,3	0.0013	1,6,4,3,9,7,10,2	0.0007
9	1,6,2,7,13,5,9,3,4	0.0007	1,6,4,3,9,7,10,2,13	0.0007
10	1,6,2,7,13,5,9,3,4,10	0.0007	1,6,4,3,9,7,10,2,13,5	0.0007
11	1,6,2,7,13,5,9,3,4,10,14	0.0007	1,6,4,3,9,7,10,2,13,5,14	0.0007
12	1,6,2,7,13,5,9,3,4,10,14,11	0.0007	1,6,4,3,9,7,10,2,13,5,14,11	0.0007
13	1,6,2,7,13,5,9,3,4,10,14,11,12	0.0007	1,6,4,3,9,7,10,2,13,5,14,11,12	0.0007
14	1,6,2,7,13,5,9,3,4,10,14,11,12,8	0.0007	1,6,4,3,9,7,10,2,13,5,14,11,12,8	0.0007

It can be observed from Table 4.17 that for a combination of 5 PMUs, the second case gives lower indicator than the first one although the first case gives better indicator for a combination of 4 PMUs. This happens because the heuristic technique places PMUs incrementally and it cannot change the PMU locations selected in previous steps when searching for new buses to be equipped with PMUs. Therefore, the heuristic placement technique can be sometimes trapped in a local optimum and does not always guarantee global optimal solution. Nevertheless, both results are comparable, and the difference is negligible.

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

In this thesis, two static state estimation algorithms are implemented which are weighted least squares "WLS" and iteratively reweighted least squares "IRLS". It is found that IRLS is a robust algorithm that can handle bad data whether single, multiple non-interacting, or multiple and interacting. It can also downweigh bad leverage measurements and suppress their effect. Three standard IEEE power systems are used to test the two algorithms which are 14-, 30-, and 118-bus systems. For various cases, IRLS outperforms WLS in rejecting bad data and providing better estimation results.

Both estimators are then extended to include current measurements into the Jacobian matrix. The current complex measurements are incorporated by their real and imaginary parts since this gives best performance in terms of convergence and estimation accuracy.

Current inclusion is a prelude to phasor measurement units "PMUs" incorporation into state estimators to form a hybrid estimator combining both SCADA and PMU measurements. Simulation results of the hybrid estimators emphasize the further improvement in estimation accuracy with both algorithms due to the presence of voltage and current phasors as compared to the previous results where there were only SCADA measurements. More specifically, IRLS hybrid estimator performs much better than WLS hybrid estimator in the presence of different bad data types. For hybrid WLS with bad data, however, placing PMUs near bad data locations is necessary for the hybrid WLS to be able to handle the bad data. Also, if the bad data is a leverage point, then only IRLS can eliminate it whereas WLS fails whether PMUs are included or not. The overall results confirm the robustness of the proposed IRLS estimator.

In the final part of the thesis, a heuristic optimal PMUs placement algorithm is demonstrated. The algorithm can work for any system size. However, it might be trapped in a local minimum for some cases. This technique helps the planning engineers in a power utility to determine the best bus locations to be equipped with PMUs when they have a limited number of PMUs.

5.2 Future Work

This section summarizes some investigations and research that may be performed as an extension to the work presented in this thesis.

- Measurement standard deviations "σ" need to be carefully specified so that the state estimation performance is evaluated in a fair way.
- Measurement errors are assumed to follow the Gaussian distribution which may not be true for practical power systems. The robust estimator IRLS needs to be enhanced so that it can handle non-Gaussian noise.
- The scale parameter "*c*" of Huber function – equation 2.9 – for IRLS algorithm is set to equal 1.5 in this work. This parameter needs to be tuned to obtain higher statistic efficiency considering different measurements and different systems.
- The main practical problem in incorporating PMUs into a SE is the high reporting rates of PMUs compared to SCADA devices. Buffering phasor measurements is

necessary to the integration of PMUs into SCADA-based state estimators. Design of an optimal buffer is a possible future research.

- The application of evolutionary algorithms and intelligence optimization techniques to solve the optimal PMU placement problem to guarantee a global optimum solution.

5.3 Thesis Publications

[1] Huthaifa Hussein and Ibrahim Habiballah, “Iteratively Reweighted Least Squares State Estimator with Phasor Measurement Units”, Electric Power Systems Research. [Submitted in Aug. 27, 2017 – Under Reviewing]

[2] MD Shoaib Shahriar, Huthaifa Hussein, and Ibrahim O. Habiballah, “Optimization of PMU Placement in Improving Power System State Estimation: Heuristic Approach vs. Genetic Algorithm”, IEEE Transactions on Power Systems. [Submitted in Nov. 7, 2017 – Under Reviewing]

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APPENDICES

A. Detailed Values of the IEEE 14-Bus System No-Bad-Data Case

	Actual	Measured	WLS	IRLS
V1	1.060000	1.059114	1.060000021	1.060000021
V2	1.045000	N/A	1.04499419	1.04499419
V3	1.010000	1.007930382	1.009502488	1.009502488
V4	1.017671	1.020368	1.017671745	1.017671745
V5	1.019514	N/A	1.019515211	1.019515211
V6	1.070000	N/A	1.071664678	1.071664678
V7	1.061520	N/A	1.061515689	1.061515689
V8	1.090000	N/A	1.089783804	1.089783804
V9	1.055932	N/A	1.055952838	1.055952838
V10	1.050985	N/A	1.051325652	1.051325652
V11	1.056907	1.060234774	1.058863517	1.058863517
V12	1.055189	N/A	1.057152043	1.057152043
V13	1.050382	1.055027229	1.052144345	1.052144345
V14	1.035530	N/A	1.03528148	1.03528148
P1	2.323933	2.341230113	2.323985187	2.323985187
P2	0.183000	0.186344158	0.187704537	0.187704537
P3	-0.942000	-0.960577582	-0.951888899	-0.951888899
P4	-0.478000	N/A	-0.472408986	-0.472408986
P5	-0.076000	N/A	-0.075318677	-0.075318677
P6	-0.112000	-0.112758021	-0.113103291	-0.113103291
P7	0.000000	N/A	-6.2058E-05	-6.2058E-05
P8	0.000000	N/A	9.71445E-17	-1.249E-16
P9	-0.295000	-0.298043472	-0.29826885	-0.29826885
P10	-0.090000	-0.089191313	-0.08937505	-0.08937505
P11	-0.035000	N/A	-0.031913367	-0.031913367
P12	-0.061000	-0.060102723	-0.060993269	-0.060993269
P13	-0.135000	-0.132318209	-0.132908415	-0.132908415
P14	-0.149000	N/A	-0.150967471	-0.150967471
Q1	-0.165493	-0.165448676	-0.165409196	-0.165409196
Q2	0.308571	0.307424263	0.310484609	0.310484609

Q3	0.060753	0.059975719	0.059460514	0.059460514
Q4	0.039000	N/A	0.040010135	0.040010135
Q5	-0.016000	N/A	-0.023137271	-0.023137271
Q6	0.052309	0.053703017	0.056300552	0.056300552
Q7	0.000000	N/A	0.001013357	0.001013357
Q8	0.176235	N/A	0.174885802	0.174885802
Q9	-0.166000	-0.160652678	-0.167426449	-0.167426449
Q10	-0.058000	-0.058259046	-0.063011086	-0.063011086
Q11	-0.018000	N/A	-0.008951386	-0.008951386
Q12	-0.016000	-0.015930788	-0.01387534	-0.01387534
Q13	-0.058000	-0.057511926	-0.053348097	-0.053348097
Q14	-0.050000	N/A	-0.056033802	-0.056033802
P1-2	1.568829	1.54358396	1.568879522	1.568879522
P1-5	0.755104	0.756843301	0.755105664	0.755105664
P2-3	0.732376	0.719566475	0.737170816	0.737170816
P2-4	0.561315	N/A	0.561293742	0.561293742
P2-5	0.415162	N/A	0.415141302	0.415141302
P3-4	-0.232857	-0.234133405	-0.238265027	-0.238265027
P4-5	-0.611582	N/A	-0.611584831	-0.611584831
P4-7	0.280742	0.281444213	0.280746404	0.280746404
P4-9	0.160798	N/A	0.160790873	0.160790873
P5-6	0.440873	N/A	0.441533844	0.441533844
P6-11	0.073533	0.072067224	0.073024925	0.073024925
P6-12	0.077861	N/A	0.078026067	0.078026067
P6-13	0.177480	0.177723984	0.177379561	0.177379561
P7-8	0.000000	0	1.31839E-16	1.17961E-16
P7-9	0.280742	N/A	0.280684346	0.280684346
P9-10	0.052276	N/A	0.049078944	0.049078944
P9-14	0.094264	0.095706876	0.094127425	0.094127425
P10-11	-0.037853	N/A	-0.040409092	-0.040409092
P12-13	0.016143	0.015826902	0.01632058	0.01632058
P13-14	0.056439	0.055630615	0.058616918	0.058616918
P2-1	-1.525853	-1.544487707	-1.525901323	-1.525901323
P5-1	-0.727475	N/A	-0.727476846	-0.727476846
P3-2	-0.709143	-0.719856934	-0.713623872	-0.713623872
P4-2	-0.544548	N/A	-0.544528256	-0.544528256

P5-2	-0.406125	N/A	-0.406104691	-0.406104691
P4-3	0.236591	N/A	0.242166823	0.242166823
P5-4	0.616727	0.618374123	0.616729015	0.616729015
P7-4	-0.280742	N/A	-0.280746404	-0.280746404
P9-4	-0.160798	N/A	-0.160790873	-0.160790873
P6-5	-0.440873	N/A	-0.441533844	-0.441533844
P11-6	-0.072979	-0.073368259	-0.072486146	-0.072486146
P12-6	-0.077143	N/A	-0.077313849	-0.077313849
P13-6	-0.175359	N/A	-0.175271526	-0.175271526
P8-7	0.000000	0	-1.38778E-16	-1.249E-16
P9-7	-0.280742	N/A	-0.280684346	-0.280684346
P10-9	-0.052147	N/A	-0.048965958	-0.048965958
P14-9	-0.093102	N/A	-0.092959616	-0.092959616
P11-10	0.037979	N/A	0.040572779	0.040572779
P13-12	-0.016080	-0.015852534	-0.016253807	-0.016253807
P14-13	-0.055898	N/A	-0.058007856	-0.058007856
Q1-2	-0.204043	-0.205567698	-0.203952787	-0.203952787
Q1-5	0.038550	0.038053937	0.038543592	0.038543592
Q2-3	0.035602	0.034850584	0.037670323	0.037670323
Q2-4	-0.015504	N/A	-0.015537722	-0.015537722
Q2-5	0.011710	N/A	0.011672618	0.011672618
Q3-4	0.044731	0.045612014	0.044160263	0.044160263
Q4-5	0.158236	N/A	0.158226033	0.158226033
Q4-7	-0.096811	-0.097777272	-0.096786821	-0.096786821
Q4-9	-0.004276	N/A	-0.004314938	-0.004314938
Q5-6	0.124707	N/A	0.117517835	0.117517835
Q6-11	0.035605	0.035262495	0.034381615	0.034381615
Q6-12	0.025034	N/A	0.023808888	0.023808888
Q6-13	0.072166	0.07277971	0.071660356	0.071660356
Q7-8	-0.171630	-0.165687539	-0.170349405	-0.170349405
Q7-9	0.057787	N/A	0.057544216	0.057544216
Q9-10	0.042191	N/A	0.039392437	0.039392437
Q9-14	0.036100	0.03719134	0.037206275	0.037206275
Q10-11	-0.016151	N/A	-0.023918785	-0.023918785
Q12-13	0.007540	0.007539443	0.008451224	0.008451224
Q13-14	0.017472	0.018110357	0.022551689	0.022551689

Q2-1	0.276762	0.282337844	0.276679391	0.276679391
Q5-1	0.022294	N/A	0.022300221	0.022300221
Q3-2	0.016022	0.015957836	0.015300251	0.015300251
Q4-2	0.030207	N/A	0.030237954	0.030237954
Q5-2	-0.020990	N/A	-0.020955632	-0.020955632
Q4-3	-0.048357	N/A	-0.047352093	-0.047352093
Q5-4	-0.142010	-0.143825936	-0.141999695	-0.141999695
Q7-4	0.113843	N/A	0.113818546	0.113818546
Q9-4	0.017323	N/A	0.017361104	0.017361104
Q6-5	-0.080495	N/A	-0.073550306	-0.073550306
Q11-6	-0.034445	-0.034194746	-0.033253343	-0.033253343
Q12-6	-0.023540	N/A	-0.022326563	-0.022326563
Q13-6	-0.067989	N/A	-0.067508976	-0.067508976
Q8-7	0.176235	0.179311933	0.174885802	0.174885802
Q9-7	-0.049766	N/A	-0.04952935	-0.04952935
Q10-9	-0.041849	N/A	-0.039092301	-0.039092301
Q14-9	-0.033629	N/A	-0.034722187	-0.034722187
Q11-10	0.016445	N/A	0.024301957	0.024301957
Q13-12	-0.007483	-0.007582494	-0.00839081	-0.00839081
Q14-13	-0.016371	N/A	-0.021311614	-0.021311614
Va1	0.000000	0.000000	0	0
Va2	-4.982589	N/A	-4.982681627	-4.982681627
Va3	-12.725100	N/A	-12.77578325	-12.77578325
Va4	-10.312901	-10.312901	-10.31292443	-10.31292443
Va5	-8.773854	N/A	-8.773882405	-8.773882405
Va6	-14.220946	N/A	-14.22065514	-14.22065514
Va7	-13.359627	N/A	-13.35970953	-13.35970953
Va8	-13.359627	N/A	-13.35970953	-13.35970953
Va9	-14.938521	N/A	-14.93825463	-14.93825463
Va10	-15.097288	N/A	-15.08762156	-15.08762156
Va11	-14.790622	N/A	-14.78916005	-14.78916005
Va12	-15.075585	N/A	-15.08213766	-15.08213766
Va13	-15.156276	N/A	-15.15400299	-15.15400299
Va14	-16.033645	N/A	-16.02431494	-16.02431494
Vpmu1	1.060000	1.060000	1.060000021	1.060000021
Vpmu2	1.045000	N/A	1.04499419	1.04499419

Vpmu3	1.010000	N/A	1.009502488	1.009502488
Vpmu4	1.017671	1.017671	1.017671745	1.017671745
Vpmu5	1.019514	N/A	1.019515211	1.019515211
Vpmu6	1.070000	N/A	1.071664678	1.071664678
Vpmu7	1.061520	N/A	1.061515689	1.061515689
Vpmu8	1.090000	N/A	1.089783804	1.089783804
Vpmu9	1.055932	N/A	1.055952838	1.055952838
Vpmu10	1.050985	N/A	1.051325652	1.051325652
Vpmu11	1.056907	N/A	1.058863517	1.058863517
Vpmu12	1.055189	N/A	1.057152043	1.057152043
Vpmu13	1.050382	N/A	1.052144345	1.052144345
Vpmu14	1.035530	N/A	1.03528148	1.03528148
IFR1-2	1.480027	1.480027	1.480074991	1.480074991
IFR1-5	0.712362	0.712362	0.71236382	0.71236382
IFR2-3	0.695231	N/A	0.69963375	0.69963375
IFR2-4	0.536402	N/A	0.536387799	0.536387799
IFR2-5	0.394810	N/A	0.394795172	0.394795172
IFR3-4	-0.234644	N/A	-0.23985252	-0.23985252
IFR4-5	-0.619090	-0.619090	-0.619090356	-0.619090356
IFR4-7	0.288441	0.288441	0.288440782	0.288440782
IFR4-9	0.156205	0.156205	0.156205302	0.156205302
IFR5-6	0.408716	N/A	0.410431736	0.410431736
IFR6-11	0.058442	N/A	0.058172238	0.058172238
IFR6-12	0.064789	N/A	0.065119517	0.065119517
IFR6-13	0.144217	N/A	0.14401915	0.14401915
IFR7-8	0.037359	N/A	0.037080544	0.037080544
IFR7-9	0.244736	N/A	0.244737081	0.244737081
IFR9-10	0.037533	N/A	0.03529114	0.03529114
IFR9-14	0.077441	N/A	0.077044497	0.077044497
IFR10-11	-0.030771	N/A	-0.031189369	-0.031189369
IFR12-13	0.012913	N/A	0.01282631	0.01282631
IFR13-14	0.047514	N/A	0.04817141	0.04817141
IFI1-2	0.192493	0.192493	0.192408286	0.192408286
IFI1-5	-0.036368	-0.036368	-0.036361878	-0.036361878
IFI2-3	-0.094810	N/A	-0.097182036	-0.097182036
IFI2-4	-0.031873	N/A	-0.031839367	-0.031839367

IFI2-5	-0.045669	N/A	-0.045632263	-0.045632263
IFI3-4	0.007584	N/A	0.009531495	0.009531495
IFI4-5	-0.045390	-0.045390	-0.045379452	-0.045379452
IFI4-7	0.044206	0.044206	0.044182039	0.044182039
IFI4-9	-0.024153	-0.024153	-0.024114083	-0.024114083
IFI5-6	-0.186850	N/A	-0.179979818	-0.179979818
IFI6-11	-0.049138	N/A	-0.047838777	-0.047838777
IFI6-12	-0.040555	N/A	-0.039421796	-0.039421796
IFI6-13	-0.106125	N/A	-0.105479787	-0.105479787
IFI7-8	0.157308	N/A	0.156134774	0.156134774
IFI7-9	-0.114074	N/A	-0.113840029	-0.113840029
IFI9-10	-0.051368	N/A	-0.048025448	-0.048025448
IFI9-14	-0.056045	N/A	-0.057022276	-0.057022276
IFI10-11	0.024218	N/A	0.031971634	0.031971634
IFI12-13	-0.010878	N/A	-0.011736044	-0.011736044
IFI13-14	-0.030103	N/A	-0.035252575	-0.035252575
ITR2-1	-1.477631	N/A	-1.477678859	-1.477678859
ITR5-1	-0.708537	N/A	-0.708538222	-0.708538222
ITR3-2	-0.688371	N/A	-0.692757144	-0.692757144
ITR4-2	-0.531762	N/A	-0.531747643	-0.531747643
ITR5-2	-0.390549	N/A	-0.39053462	-0.39053462
ITR4-3	0.237234	N/A	0.242447243	0.242447243
ITR5-4	0.619090	N/A	0.619090356	0.619090356
ITR7-4	-0.282095	N/A	-0.282095085	-0.282095085
ITR9-4	-0.151363	N/A	-0.151362938	-0.151362938
ITR6-5	-0.380924	N/A	-0.382522378	-0.382522378
ITR11-6	-0.058442	N/A	-0.058172238	-0.058172238
ITR12-6	-0.064789	N/A	-0.065119517	-0.065119517
ITR13-6	-0.144217	N/A	-0.14401915	-0.14401915
ITR8-7	-0.037359	N/A	-0.037080544	-0.037080544
ITR9-7	-0.244736	N/A	-0.244737081	-0.244737081
ITR10-9	-0.037533	N/A	-0.03529114	-0.03529114
ITR14-9	-0.077441	N/A	-0.077044497	-0.077044497
ITR11-10	0.030771	N/A	0.031189369	0.031189369
ITR13-12	-0.012913	N/A	-0.01282631	-0.01282631
ITR14-13	-0.047514	N/A	-0.04817141	-0.04817141

ITI2-1	-0.137026	N/A	-0.136940694	-0.136940694
ITI5-1	0.087230	N/A	0.087224466	0.087224466
ITI3-2	0.139185	N/A	0.141541697	0.141541697
ITI4-2	0.066591	N/A	0.06655806	0.06655806
ITI5-2	0.081110	N/A	0.081073562	0.081073562
ITI4-3	0.005129	N/A	0.003177248	0.003177248
ITI5-4	0.045390	N/A	0.045379452	0.045379452
ITI7-4	-0.043234	N/A	-0.043210034	-0.043210034
ITI9-4	0.023404	N/A	0.023366546	0.023366546
ITI6-5	0.174144	N/A	0.16774119	0.16774119
ITI11-6	0.049138	N/A	0.047838777	0.047838777
ITI12-6	0.040555	N/A	0.039421796	0.039421796
ITI13-6	0.106125	N/A	0.105479787	0.105479787
ITI8-7	-0.157308	N/A	-0.156134774	-0.156134774
ITI9-7	0.114074	N/A	0.113840029	0.113840029
ITI10-9	0.051368	N/A	0.048025448	0.048025448
ITI14-9	0.056045	N/A	0.057022276	0.057022276
ITI11-10	-0.024218	N/A	-0.031971634	-0.031971634
ITI13-12	0.010878	N/A	0.011736044	0.011736044
ITI14-13	0.030103	N/A	0.035252575	0.035252575

B. Detailed Values of the IEEE 30-Bus System No-Bad-Data Case

	Actual	**Measured**	**WLS**	**IRLS**
V1	1.06	1.054683954	1.059998638	1.05999821
V2	1.045	N/A	1.0450158	1.044994281
V3	1.021177684	1.023549647	1.021176744	1.021177361
V4	1.01230043	1.013318757	1.01229497	1.012299307
V5	1.01	1.01763247	1.009645603	1.009471852
V6	1.010625749	N/A	1.010432141	1.010017726
V7	1.002597084	N/A	1.002198783	1.001880526
V8	1.01	1.00099831	1.00966573	1.009271152
V9	1.051131712	N/A	1.051716615	1.051531131
V10	1.045378954	1.049070049	1.046471862	1.046379145
V11	1.082	N/A	1.082414113	1.08229387
V12	1.05733893	1.055663308	1.057339032	1.057338954
V13	1.071	N/A	1.071000761	1.071000669

V14	1.042507806	N/A	1.042511617	1.042511456
V15	1.037915881	N/A	1.037920592	1.037920426
V16	1.044625844	N/A	1.044628628	1.044628486
V17	1.040150253	N/A	1.041138852	1.041073738
V18	1.028396289	1.029406662	1.02899913	1.028981989
V19	1.025899933	N/A	1.02669025	1.026663804
V20	1.02998673	N/A	1.030870733	1.030840531
V21	1.032982194	1.043105492	1.034442364	1.034379331
V22	1.033513632	N/A	1.035009521	1.034952501
V23	1.027428984	N/A	1.026897057	1.026914029
V24	1.021845772	1.025164589	1.021846756	1.021846724
V25	1.017618614	1.009789436	1.017619831	1.017619831
V26	0.999946403	1.00223775	1.001600874	1.001600874
V27	1.023538511	N/A	1.023240632	1.023165381
V28	1.007101046	1.005204198	1.006256106	1.006061004
V29	1.003705785	1.004230275	1.0028131	1.0027364
V30	0.992234799	N/A	0.9914617	0.991384061
P1	2.609569476	2.610264801	2.609440884	2.609438899
P2	0.183	0.182217506	0.183068657	0.183003622
P3	-0.024	N/A	-0.02398654	-0.02399841
P4	-0.076	-0.075093714	-0.07202235	-0.07201593
P5	-0.942	-0.936570365	-0.93815558	-0.9381155
P6	0	N/A	-0.00916139	-0.00926062
P7	-0.228	-0.230591493	-0.23257182	-0.23257827
P8	-0.3	N/A	-0.29674488	-0.29679616
P9	0	0	3.42477E-07	3.37844E-07
P10	-0.058	-0.05712015	-0.0568093	-0.05683546
P11	0	N/A	3.42477E-07	3.37844E-07
P12	-0.112	N/A	-0.11299852	-0.11297463
P13	0	N/A	2.49584E-06	2.44055E-06
P14	-0.062	-0.061341427	-0.06200693	-0.06200684
P15	-0.082	-0.081327484	-0.08188189	-0.08188368
P16	-0.035	-0.034884434	-0.03641532	-0.03647787
P17	-0.09	N/A	-0.08927085	-0.08925888
P18	-0.032	-0.03216159	-0.03200754	-0.03200753
P19	-0.095	-0.094369324	-0.09500669	-0.09500779

P20	-0.022	N/A	-0.02206747	-0.02208661
P21	-0.175	-0.177965115	-0.17667027	-0.17667395
P22	0	N/A	0.003250099	0.003207661
P23	-0.032	N/A	-0.03438723	-0.03437068
P24	-0.087	-0.085604788	-0.08769748	-0.08766746
P25	0	N/A	5.32337E-05	0.000105182
P26	-0.035	N/A	-0.03465425	-0.03465425
P27	0	N/A	0.001004374	0.000956446
P28	0	N/A	0.00523611	0.005471622
P29	-0.024	-0.023614171	-0.02558067	-0.02558043
P30	-0.106	-0.105707851	-0.10636382	-0.1063637
Q1	-0.204178835	-0.206368581	-0.20446015	-0.20408766
Q2	0.433694618	0.443490431	0.437086228	0.439614464
Q3	-0.012	N/A	-0.0118811	-0.01196997
Q4	-0.016	-0.015420161	-0.01235227	-0.00194039
Q5	0.166587907	0.165931032	0.163492043	0.163984116
Q6	0	N/A	0.007745109	-0.01185964
Q7	-0.109	-0.1108636	-0.11013845	-0.11011547
Q8	0.061112665	N/A	0.059349877	0.058860815
Q9	0	0	3.63277E-07	3.93522E-07
Q10	-0.02	-0.02069799	-0.01928329	-0.01909456
Q11	0.16057446	N/A	0.159747137	0.160068862
Q12	-0.075	N/A	-0.07495638	-0.07497661
Q13	0.104507186	N/A	0.104512306	0.104512184
Q14	-0.016	-0.016293625	-0.01598536	-0.01598568
Q15	-0.025	-0.025516923	-0.0251016	-0.02510901
Q16	-0.018	-0.018047852	-0.02295445	-0.02258578
Q17	-0.058	N/A	-0.05415635	-0.05416073
Q18	-0.009	-0.008734453	-0.00768074	-0.00768663
Q19	-0.034	-0.034800576	-0.03398011	-0.03399471
Q20	-0.007	N/A	-0.00662503	-0.00636201
Q21	-0.112	-0.11061716	-0.10794084	-0.1077795
Q22	0	N/A	0.011058994	0.011267474
Q23	-0.016	N/A	-0.01957	-0.01942678
Q24	-0.067	-0.066581838	-0.07285696	-0.07261583
Q25	0	N/A	-0.00291818	-0.00257719

Q26	-0.023	N/A	-0.01893247	-0.01893247
Q27	0	N/A	0.001520934	0.001509626
Q28	0	N/A	-0.01631626	-0.01200141
Q29	-0.009	N/A	-0.00982692	-0.00982649
Q30	-0.019	-0.019005207	-0.01925137	-0.01925116
P1-2	1.733071463	N/A	1.732946293	1.732944427
P1-3	0.876498012	0.870690448	0.876494591	0.876494472
P2-4	0.436526909	0.439450587	0.436580035	0.436557877
P3-4	0.821419012	N/A	0.821429224	0.821417251
P2-5	0.823613183	0.819852249	0.822756563	0.822813312
P2-6	0.603799781	N/A	0.604551967	0.60445336
P4-6	0.721273254	0.729122851	0.724292774	0.72429141
P5-7	-0.147817118	-0.147421588	-0.14477628	-0.14468826
P6-7	0.381321171	0.386966853	0.382832546	0.382767111
P6-8	0.295630561	0.291664633	0.291955751	0.291922718
P6-9	0.277212428	0.271700275	0.277836145	0.277895599
P6-10	0.158396598	0.159571336	0.15876303	0.158793273
P9-11	0	0	-3.4248E-07	-3.3784E-07
P9-10	0.277212428	N/A	0.27783683	0.277896275
P4-12	0.441932236	N/A	0.442950659	0.442926132
P12-13	0	0	-2.4958E-06	-2.4405E-06
P12-14	0.078575198	0.079978381	0.078580836	0.078580709
P12-15	0.178917656	N/A	0.178927656	0.178927293
P12-16	0.072439382	0.072667404	0.072446144	0.072445945
P14-15	0.015831928	0.015643349	0.015830634	0.015830592
P16-17	0.03690104	N/A	0.035492496	0.035429745
P15-18	0.060168286	0.059302573	0.059397157	0.059403205
P18-19	0.02778235	0.02807546	0.02702008	0.027025851
P19-20	-0.067266584	N/A	-0.06803221	-0.06802761
P10-20	0.090253503	N/A	0.091108014	0.091120697
P10-17	0.053316688	0.053553712	0.053990284	0.054040398
P10-21	0.157855887	N/A	0.158432555	0.158434848
P10-22	0.076182948	N/A	0.076259706	0.07625814
P21-22	-0.018256759	N/A	-0.01932163	-0.01932065
P15-23	0.050353298	N/A	0.051251385	0.051243147
P22-24	0.057393475	0.057962678	0.059671336	0.059629606

P23-24	0.018039446	0.017936631	0.016528861	0.016537738
P24-25	-0.012082695	-0.011843811	-0.0120833	-0.01208338
P25-26	0.035446265	0.035678732	0.035049685	0.035049685
P25-27	-0.047628454	-0.048042049	-0.04717924	-0.04712738
P28-27	0.180688827	0.179296708	0.181245877	0.181241342
P27-29	0.061899457	N/A	0.063219631	0.063219606
P27-30	0.070920029	0.072237875	0.07161579	0.071615898
P29-30	0.037037146	0.037061343	0.036735931	0.036736013
P8-28	-0.005449594	N/A	-0.00583762	-0.00592328
P6-28	0.186734987	N/A	0.182429674	0.182270373
P2-1	-1.680939874	-1.653618314	-1.68081991	-1.68082093
P3-1	-0.84541901	N/A	-0.84541576	-0.84541566
P4-2	-0.426342098	N/A	-0.4263923	-0.4263718
P4-3	-0.812863391	-0.801427462	-0.81287348	-0.81286168
P5-2	-0.794182882	N/A	-0.7933793	-0.79342724
P6-2	-0.58434098	-0.57964541	-0.58504155	-0.58493965
P6-4	-0.714954765	N/A	-0.71793698	-0.71797004
P7-5	0.149510476	N/A	0.146420298	0.146345476
P7-6	-0.377510476	N/A	-0.37899212	-0.37892374
P8-6	-0.294550406	N/A	-0.29090725	-0.29087289
P9-6	-0.277212428	N/A	-0.27783614	-0.2778956
P10-6	-0.158396598	N/A	-0.15876303	-0.15879327
P11-9	0	N/A	3.42477E-07	3.37844E-07
P10-9	-0.277212428	-0.279606552	-0.27783683	-0.27789628
P12-4	-0.441932236	-0.449734001	-0.44295066	-0.44292613
P13-12	0	N/A	2.49584E-06	2.44055E-06
P14-12	-0.077831928	N/A	-0.07783756	-0.07783744
P15-12	-0.176749105	-0.17334518	-0.17675923	-0.17675887
P16-12	-0.07190104	N/A	-0.07190782	-0.07190762
P15-14	-0.015772479	N/A	-0.0157712	-0.01577116
P17-16	-0.036825761	N/A	-0.03542778	-0.0353649
P18-15	-0.05978235	N/A	-0.05902762	-0.05903338
P19-18	-0.027733416	N/A	-0.02697447	-0.02698018
P20-19	0.067437961	0.068705333	0.068208648	0.068203931
P20-10	-0.089437961	-0.090187366	-0.09027612	-0.09029054
P17-10	-0.053174239	N/A	-0.05384307	-0.05389398

P21-10	-0.156743241	-0.156444277	-0.15734864	-0.1573533
P22-10	-0.075656077	N/A	-0.07574947	-0.07574929
P22-21	0.018262602	0.018398218	0.019328236	0.019327345
P23-15	-0.050039446	-0.050723802	-0.05091609	-0.05090842
P24-22	-0.056937918	-0.056806941	-0.05913506	-0.05909629
P24-23	-0.017979387	-0.017658651	-0.01647912	-0.01648779
P25-24	0.012182188	N/A	0.012182789	0.012182874
P26-25	-0.035	N/A	-0.03465425	-0.03465425
P27-25	0.047869341	N/A	0.04741483	0.047362284
P27-28	-0.180688827	-0.184171291	-0.18124588	-0.18124134
P29-27	-0.061037146	N/A	-0.0623166	-0.06231644
P30-27	-0.069298252	N/A	-0.06995821	-0.06995806
P30-29	-0.036701748	-0.036729561	-0.03640561	-0.03640564
P28-8	0.005468181	0.005381177	0.005862466	0.005945916
P28-6	-0.186157009	N/A	-0.18187223	-0.18171564
Q1-2	-0.247027655	N/A	-0.2473067	-0.24692766
Q1-3	0.04284882	0.042070591	0.042846551	0.042840002
Q2-4	0.047495716	0.048110799	0.047609482	0.047460052
Q3-4	-0.038546165	N/A	-0.03842901	-0.03852435
Q2-5	0.027816529	0.026762578	0.02979001	0.030568165
Q2-6	0.013723967	N/A	0.01476567	0.01705165
Q4-6	-0.159119884	-0.158275626	-0.15532524	-0.1451713
Q5-7	0.114903172	0.114913166	0.113989333	0.115214287
Q6-7	-0.02781358	-0.028355175	-0.02576885	-0.02695184
Q6-8	-0.071964993	-0.073615377	-0.06759441	-0.0680629
Q6-9	-0.080929856	-0.082163068	-0.08477105	-0.08590868
Q6-10	0.001865412	N/A	-0.00053393	-0.00115333
Q9-11	-0.155993444	-0.1515839	-0.15521667	-0.15551912
Q9-10	0.058819046	N/A	0.054003735	0.053110773
Q4-12	0.144099996	N/A	0.144175099	0.144193169
Q12-13	-0.103174152	-0.102281452	-0.10317914	-0.10317902
Q12-14	0.024003054	0.023724315	0.023985242	0.023985638
Q12-15	0.067899201	N/A	0.067857294	0.067858178
Q12-16	0.033485672	0.033523906	0.033468341	0.033468771
Q14-15	0.006457945	0.006338871	0.006454767	0.006454841
Q16-17	0.014353731	N/A	0.00938198	0.009751082

Q15-18	0.015953008	0.015338009	0.013495177	0.013572495
Q18-19	0.006167108	0.005930425	0.005061925	0.005132783
Q19-20	-0.027931832	N/A	-0.0290104	-0.02895427
Q10-20	0.037095615	N/A	0.037845835	0.037522577
Q10-17	0.044294043	0.044884007	0.045395801	0.045029476
Q10-21	0.100108547	N/A	0.094912568	0.094483582
Q10-22	0.046000257	N/A	0.043246443	0.042991894
Q21-22	-0.014286201	N/A	-0.01536119	-0.01562374
Q15-23	0.029078833	N/A	0.031390237	0.031306467
Q22-24	0.030615815	0.029532699	0.03787876	0.037572812
Q23-24	0.012444851	0.012479737	0.011142947	0.011203537
Q24-25	0.020127991	0.019575665	0.020127627	0.020127577
Q25-26	0.023666591	0.023349035	0.01952313	0.01952313
Q25-27	-0.003712357	-0.003823298	-0.00248744	-0.0021465
Q28-27	0.050360146	0.051866424	0.048858112	0.048519677
Q27-29	0.016687887	N/A	0.017460883	0.017460855
Q27-30	0.016627758	0.017281138	0.017067797	0.017068003
Q29-30	0.006058597	0.006025452	0.005927733	0.005927887
Q8-28	-0.005446281	N/A	-0.00273248	-0.00370206
Q6-28	0.001146799	N/A	0.013258554	0.00959178
Q2-1	0.344658406	0.336289292	0.344921066	0.344534597
Q3-1	0.026546169	N/A	0.026547902	0.026554377
Q4-2	-0.055407543	N/A	-0.05551281	-0.05536775
Q4-3	0.054427432	0.052476006	0.054310692	0.054405485
Q5-2	0.051684736	N/A	0.04950271	0.048769829
Q6-2	0.005801891	0.006021016	0.0049235	0.002664017
Q6-4	0.171894329	N/A	0.168231289	0.157961317
Q7-5	-0.131291035	N/A	-0.13048617	-0.13166776
Q7-6	0.022291035	N/A	0.020347724	0.021552288
Q8-6	0.066558946	N/A	0.062082352	0.062562879
Q9-6	0.097174399	N/A	0.101213302	0.102408741
Q10-6	0.010960657	N/A	0.013422706	0.01405814
Q11-9	0.16057446	N/A	0.159747137	0.160068862
Q10-9	-0.050823859	-0.052073506	-0.046037	-0.04514746
Q12-4	-0.097213775	-0.100417826	-0.09708812	-0.09711018
Q13-12	0.104507186	N/A	0.104512306	0.104512184

Q14-12	-0.022457945	N/A	-0.02244013	-0.02244052
Q15-12	-0.063627615	-0.062877837	-0.06358595	-0.06358684
Q16-12	-0.032353731	N/A	-0.03233643	-0.03233686
Q15-14	-0.006404226	N/A	-0.00640106	-0.00640114
Q17-16	-0.014077466	N/A	-0.00914448	-0.00951312
Q18-15	-0.015167108	N/A	-0.01274266	-0.01281941
Q19-18	-0.006068168	N/A	-0.00496971	-0.00504044
Q20-19	0.028274586	0.028208533	0.02936327	0.029306911
Q20-10	-0.035274586	-0.036317015	-0.0359883	-0.03566892
Q17-10	-0.043922534	N/A	-0.04501187	-0.04464761
Q21-10	-0.097713799	-0.095658924	-0.09257965	-0.09215576
Q22-10	-0.044913902	N/A	-0.0421944	-0.0419427
Q22-21	0.014298087	0.014494728	0.015374629	0.01563736
Q23-15	-0.028444851	-0.028477369	-0.03071294	-0.03063032
Q24-22	-0.02990673	-0.030782653	-0.03704404	-0.03674269
Q24-23	-0.012322003	-0.012398239	-0.0110412	-0.01110138
Q25-24	-0.019954234	N/A	-0.01995387	-0.01995382
Q26-25	-0.023	N/A	-0.01893247	-0.01893247
Q27-25	0.004172313	N/A	0.00293728	0.002595035
Q27-28	-0.037487958	-0.037877634	-0.03594503	-0.03561427
Q29-27	-0.015058597	N/A	-0.01575466	-0.01575438
Q30-27	-0.013575151	N/A	-0.01394779	-0.01394753
Q30-29	-0.005424849	-0.005448178	-0.00530358	-0.00530364
Q28-8	-0.038030412	-0.039456872	-0.04067368	-0.03968559
Q28-6	-0.012329734	N/A	-0.02450069	-0.02083549
Va1	0	0	0	0
Va2	-5.378243009	N/A	-5.37807288	-5.37780249
Va3	-7.528659574	-7.528659574	-7.52865172	-7.528665
Va4	-9.279432387	N/A	-9.2793713	-9.27942065
Va5	-14.1487671	N/A	-14.1371127	-14.137146
Va6	-11.0550233	N/A	-11.0597873	-11.0537821
Va7	-12.85231876	N/A	-12.862072	-12.8588756
Va8	-11.79738537	N/A	-11.7909241	-11.7857454
Va9	-14.09796901	N/A	-14.1084728	-14.1049116
Va10	-15.68817315	N/A	-15.6997091	-15.6969103
Va11	-14.09796901	N/A	-14.1084692	-14.104908

Va12	-14.9329077	-14.9329077	-14.9459476	-14.9456582
Va13	-14.9329077	N/A	-14.94593	-14.945641
Va14	-15.82452199	N/A	-15.8377476	-15.8374542
Va15	-15.91636332	N/A	-15.9296117	-15.929317
Va16	-15.5154241	N/A	-15.5286171	-15.5283237
Va17	-15.84994785	N/A	-15.8622774	-15.8603502
Va18	-16.53018883	N/A	-16.548184	-16.5475255
Va19	-16.70372229	N/A	-16.7199711	-16.7191148
Va20	-16.50719249	N/A	-16.5229278	-16.5219743
Va21	-16.13066683	N/A	-16.1529737	-16.151042
Va22	-16.11643737	N/A	-16.1381073	-16.1363382
Va23	-16.30662593	N/A	-16.3174014	-16.3174613
Va24	-16.48278713	-16.48278713	-16.4807672	-16.4805185
Va25	-16.05455911	N/A	-16.052533	-16.0522833
Va26	-16.47398097	N/A	-16.5220463	-16.5217966
Va27	-15.53008003	N/A	-15.5256934	-15.5239499
Va28	-11.67729672	N/A	-11.6566365	-11.6539535
Va29	-16.75931301	N/A	-16.7775108	-16.7759549
Va30	-17.64161308	N/A	-17.655225	-17.653805
Vpmu1	1.06	1.06	1.059998638	1.05999821
Vpmu2	1.045	N/A	1.0450158	1.044994281
Vpmu3	1.021177684	1.021177684	1.021176744	1.021177361
Vpmu4	1.01230043	N/A	1.01229497	1.012299307
Vpmu5	1.01	N/A	1.009645603	1.009471852
Vpmu6	1.010625749	N/A	1.010432141	1.010017726
Vpmu7	1.002597084	N/A	1.002198783	1.001880526
Vpmu8	1.01	N/A	1.00966573	1.009271152
Vpmu9	1.051131712	N/A	1.051716615	1.051531131
Vpmu10	1.045378954	N/A	1.046471862	1.046379145
Vpmu11	1.082	N/A	1.082414113	1.08229387
Vpmu12	1.05733893	1.05733893	1.057339032	1.057338954
Vpmu13	1.071	N/A	1.071000761	1.071000669
Vpmu14	1.042507806	N/A	1.042511617	1.042511456
Vpmu15	1.037915881	N/A	1.037920592	1.037920426
Vpmu16	1.044625844	N/A	1.044628628	1.044628486
Vpmu17	1.040150253	N/A	1.041138852	1.041073738

Vpmu18	1.028396289	N/A	1.02899913	1.028981989
Vpmu19	1.025899933	N/A	1.02669025	1.026663804
Vpmu20	1.02998673	N/A	1.030870733	1.030840531
Vpmu21	1.032982194	N/A	1.034442364	1.034379331
Vpmu22	1.033513632	N/A	1.035009521	1.034952501
Vpmu23	1.027428984	N/A	1.026897057	1.026914029
Vpmu24	1.021845772	1.021845772	1.021846756	1.021846724
Vpmu25	1.017618614	N/A	1.017619831	1.017619831
Vpmu26	0.999946403	N/A	1.001600874	1.001600874
Vpmu27	1.023538511	N/A	1.023240632	1.023165381
Vpmu28	1.007101046	N/A	1.006256106	1.006061004
Vpmu29	1.003705785	N/A	1.0028131	1.0027364
Vpmu30	0.992234799	N/A	0.9914617	0.991384061
IFR1-2	1.634973079	1.634973079	1.634857093	1.634855994
IFR1-3	0.826884917	0.826884917	0.826882752	0.826882974
IFR2-4	0.411630019	N/A	0.411664456	0.411665623
IFR3-4	0.802395448	0.802395448	0.802391079	0.802391188
IFR2-5	0.782181896	N/A	0.781177256	0.781178101
IFR2-6	0.57402419	N/A	0.574638896	0.574352083
IFR4-6	0.728531193	N/A	0.730874443	0.72925259
IFR5-7	-0.169722737	N/A	-0.1666255	-0.16686605
IFR6-7	0.37558758	N/A	0.376735568	0.377056119
IFR6-8	0.300748414	N/A	0.296408096	0.296585476
IFR6-9	0.284563067	N/A	0.285954826	0.286342778
IFR6-10	0.153468892	N/A	0.154307082	0.154520469
IFR9-11	0.03614864	N/A	0.035974559	0.036042104
IFR9-10	0.242154039	N/A	0.243689492	0.244001353
IFR4-12	0.40789561	N/A	0.40887896	0.408850237
IFR12-13	0.025144935	0.025144935	0.025165327	0.025164874
IFR12-14	0.06595452	0.06595452	0.065954657	0.065954658
IFR12-15	0.146952403	0.146952403	0.1469477	0.146947696
IFR12-16	0.058036383	0.058036383	0.058035801	0.058035762
IFR14-15	0.01292162	N/A	0.012918879	0.012918874
IFR16-17	0.030361798	N/A	0.030331514	0.030179139
IFR15-18	0.051532848	N/A	0.051461015	0.051446327
IFR18-19	0.024192459	N/A	0.023769872	0.023756172

IFR19-20	-0.054976071	N/A	-0.05533299	-0.05534654
IFR10-20	0.073524174	N/A	0.074027928	0.074132589
IFR10-17	0.037645035	N/A	0.037929518	0.038076423
IFR10-21	0.1194838	N/A	0.121206327	0.121336362
IFR10-22	0.058262509	N/A	0.058971866	0.059044373
IFR21-22	-0.01313564	N/A	-0.01380969	-0.01373967
IFR15-23	0.038970889	N/A	0.039182251	0.039196995
IFR22-24	0.045126868	N/A	0.045208685	0.045256183
IFR23-24	0.013450597	N/A	0.012398883	0.012390381
IFR24-25	-0.016927229	-0.016927229	-0.01692713	-0.01692713
IFR25-26	0.027042313	N/A	0.0277948	0.027794922
IFR25-27	-0.043969542	N/A	-0.0438787	-0.04392243
IFR28-27	0.165580476	N/A	0.166594041	0.166693817
IFR27-29	0.053902636	N/A	0.054961673	0.054966702
IFR27-30	0.062409722	N/A	0.062970546	0.062976285
IFR29-30	0.03359248	N/A	0.033367246	0.033370273
IFR8-28	-0.004179189	N/A	-0.00510673	-0.00499593
IFR6-28	0.181125345	N/A	0.174675832	0.175293748
IFI1-2	0.233044958	0.233044958	0.233308505	0.232951015
IFI1-3	-0.040423415	-0.040423415	-0.04042133	-0.04041516
IFI2-4	-0.084404213	N/A	-0.08451487	-0.08437031
IFI3-4	-0.067970706	-0.067970706	-0.06808571	-0.06799176
IFI2-5	-0.100374688	N/A	-0.10217417	-0.10291905
IFI2-6	-0.067232402	N/A	-0.06828959	-0.07045736
IFI4-6	0.040237655	N/A	0.036058171	0.026158293
IFI5-7	-0.074539629	N/A	-0.07445824	-0.07566896
IFI6-7	-0.045339719	N/A	-0.04765262	-0.04647072
IFI6-8	0.013795354	N/A	0.010225533	0.010722323
IFI6-9	0.025995928	N/A	0.029589758	0.030726048
IFI6-10	-0.031865008	N/A	-0.02962298	-0.02902281
IFI9-11	0.143935362	N/A	0.143132472	0.143438908
IFI9-10	-0.118511345	N/A	-0.1141941	-0.11338927
IFI4-12	-0.210881697	N/A	-0.21111784	-0.211131
IFI12-13	0.094283655	0.094283655	0.09428309	0.094283102
IFI12-14	-0.041084553	-0.041084553	-0.04108466	-0.04108466
IFI12-15	-0.105652991	-0.105652991	-0.10565057	-0.10565055

IFI12-16	-0.048254675	-0.048254675	-0.04825369	-0.04825374
IFI14-15	-0.010101065	N/A	-0.01010074	-0.01010073
IFI16-17	-0.022689086	N/A	-0.01774941	-0.01807362
IFI15-18	-0.030678402	N/A	-0.02820917	-0.02828214
IFI18-19	-0.013435355	N/A	-0.01219453	-0.01226207
IFI19-20	0.044923642	N/A	0.046125293	0.046072007
IFI10-20	-0.05750873	N/A	-0.05837457	-0.058082
IFI10-17	-0.054583944	N/A	-0.05572224	-0.05540125
IFI10-21	-0.133027133	N/A	-0.12828135	-0.1278927
IFI10-22	-0.062069967	N/A	-0.05950336	-0.05927115
IFI21-22	0.018195876	N/A	0.019459857	0.01970412
IFI15-23	-0.040246632	N/A	-0.04263438	-0.0425544
IFI22-24	-0.043874091	N/A	-0.0511802	-0.05088645
IFI23-24	-0.016555212	N/A	-0.01493629	-0.01499511
IFI24-25	-0.015533304	-0.015533304	-0.01553338	-0.01553338
IFI25-26	-0.031982829	N/A	-0.02796111	-0.02796099
IFI25-27	0.016449525	N/A	0.01516911	0.014832852
IFI28-27	-0.08528353	N/A	-0.08394532	-0.08362342
IFI27-29	-0.031900935	N/A	-0.03297931	-0.03298003
IFI27-30	-0.034203997	N/A	-0.03480553	-0.0348064
IFI29-30	-0.016420137	N/A	-0.01623379	-0.01623429
IFI8-28	0.006381599	N/A	0.003830659	0.004789459
IFI6-28	-0.036543944	N/A	-0.04751272	-0.04392061
ITR2-1	-1.632387248	N/A	-1.63227131	-1.63227039
ITR3-1	-0.824155462	N/A	-0.8241533	-0.82415352
ITR4-2	-0.406824286	N/A	-0.40685879	-0.40686005
ITR4-3	-0.801147923	N/A	-0.80114356	-0.80114366
ITR5-2	-0.774974888	N/A	-0.77397625	-0.77397812
ITR6-2	-0.568568704	N/A	-0.56918259	-0.56889934
ITR6-4	-0.726924594	N/A	-0.72926765	-0.72764661
ITR7-5	0.174515731	N/A	0.171416374	0.171655217
ITR7-6	-0.372044732	N/A	-0.37319167	-0.37351485
ITR8-6	-0.298947132	N/A	-0.29460742	-0.29478639
ITR9-6	-0.278302679	N/A	-0.27966382	-0.28004324
ITR10-6	-0.148711357	N/A	-0.14952356	-0.14973033
ITR11-9	-0.03614864	N/A	-0.03597456	-0.0360421

ITR10-9	-0.242154039	N/A	-0.24368949	-0.24400135
ITR12-4	-0.380158709	N/A	-0.38107519	-0.38104842
ITR13-12	-0.025144935	N/A	-0.02516533	-0.02516487
ITR14-12	-0.06595452	N/A	-0.06595466	-0.06595466
ITR15-12	-0.146952403	N/A	-0.1469477	-0.1469477
ITR16-12	-0.058036383	N/A	-0.0580358	-0.05803576
ITR15-14	-0.01292162	N/A	-0.01291888	-0.01291887
ITR17-16	-0.030361798	N/A	-0.03033151	-0.03017914
ITR18-15	-0.051532848	N/A	-0.05146101	-0.05144633
ITR19-18	-0.024192459	N/A	-0.02376987	-0.02375617
ITR20-19	0.054976071	N/A	0.055332986	0.055346535
ITR20-10	-0.073524174	N/A	-0.07402793	-0.07413259
ITR17-10	-0.037645035	N/A	-0.03792952	-0.03807642
ITR21-10	-0.1194838	N/A	-0.12120633	-0.12133636
ITR22-10	-0.058262509	N/A	-0.05897187	-0.05904437
ITR22-21	0.01313564	N/A	0.013809693	0.013739674
ITR23-15	-0.038970889	N/A	-0.03918225	-0.03919699
ITR24-22	-0.045126868	N/A	-0.04520869	-0.04525618
ITR24-23	-0.013450597	N/A	-0.01239888	-0.01239038
ITR25-24	0.016927229	N/A	0.016927128	0.016927126
ITR26-25	-0.027042313	N/A	-0.0277948	-0.02779492
ITR27-25	0.043969542	N/A	0.043878696	0.043922427
ITR27-28	-0.1602819	N/A	-0.16126303	-0.16135961
ITR29-27	-0.053902636	N/A	-0.05496167	-0.0549667
ITR30-27	-0.062409722	N/A	-0.06297055	-0.06297628
ITR30-29	-0.03359248	N/A	-0.03336725	-0.03337027
ITR28-8	0.012960303	N/A	0.01387273	0.013756465
ITR28-6	-0.178540778	N/A	-0.17209439	-0.17271406
ITI2-1	-0.177594411	N/A	-0.17785757	-0.17750065
ITI3-1	0.082699857	N/A	0.082697721	0.082691563
ITI4-2	0.121930142	N/A	0.122040994	0.121896128
ITI4-3	0.076418702	N/A	0.07653368	0.076439747
ITI5-2	0.142587682	N/A	0.144381369	0.145122286
ITI6-2	0.105235881	N/A	0.10628951	0.108449666
ITI6-4	-0.031278492	N/A	-0.02709996	-0.0172018
ITI7-5	0.094499393	N/A	0.094410662	0.095616629

ITI7-6	0.0620792	N/A	0.064386721	0.063199007
ITI8-6	-0.004882935	N/A	-0.00131541	-0.00181559
ITI9-6	-0.025424017	N/A	-0.02893878	-0.03005008
ITI10-6	0.030877192	N/A	0.028704665	0.028123101
ITI11-9	-0.143935362	N/A	-0.14313247	-0.14343891
ITI10-9	0.118511345	N/A	0.114194103	0.113389274
ITI12-4	0.196541741	N/A	0.19676183	0.196774093
ITI13-12	-0.094283655	N/A	-0.09428309	-0.0942831
ITI14-12	0.041084553	N/A	0.041084658	0.041084659
ITI15-12	0.105652991	N/A	0.105650568	0.105650553
ITI16-12	0.048254675	N/A	0.04825369	0.048253744
ITI15-14	0.010101065	N/A	0.010100738	0.010100731
ITI17-16	0.022689086	N/A	0.017749414	0.018073616
ITI18-15	0.030678402	N/A	0.028209169	0.02828214
ITI19-18	0.013435355	N/A	0.012194534	0.012262071
ITI20-19	-0.044923642	N/A	-0.04612529	-0.04607201
ITI20-10	0.05750873	N/A	0.058374573	0.058082
ITI17-10	0.054583944	N/A	0.05572224	0.05540125
ITI21-10	0.133027133	N/A	0.128281349	0.127892704
ITI22-10	0.062069967	N/A	0.059503359	0.059271151
ITI22-21	-0.018195876	N/A	-0.01945986	-0.01970412
ITI23-15	0.040246632	N/A	0.04263438	0.042554397
ITI24-22	0.043874091	N/A	0.051180197	0.050886446
ITI24-23	0.016555212	N/A	0.014936286	0.014995107
ITI25-24	0.015533304	N/A	0.015533378	0.015533381
ITI26-25	0.031982829	N/A	0.027961115	0.027960994
ITI27-25	-0.016449525	N/A	-0.01516911	-0.01483285
ITI27-28	0.082554457	N/A	0.081259069	0.080947474
ITI29-27	0.031900935	N/A	0.032979314	0.032980032
ITI30-27	0.034203997	N/A	0.034805531	0.034806398
ITI30-29	0.016420137	N/A	0.016233787	0.016234293
ITI28-8	0.035881746	N/A	0.038410045	0.037439492
ITI28-6	0.049401784	N/A	0.060364318	0.056768512

C. Measured Values of the IEEE 118-Bus System No-Bad-Data Case

	Actual	Measured	WLS	IRLS

V2	0.971392795	0.9687	0.97139218	0.971392149
V3	0.967691944	0.9694	0.96763911	0.967638249
V4	0.998	0.9888	0.99797024	0.997966746
V5	1.001984637	0.9945	1.00200066	1.001996442
V9	1.042918205	1.0463	1.0429192	1.042919526
V12	0.99	0.9828	0.98999973	0.990002366
V15	0.97	0.9703	0.96982714	0.969826148
V17	0.995088532	1.0000	0.99508668	0.995086219
V18	0.973	0.9768	0.97298604	0.972981347
V21	0.957724895	0.9604	0.95896672	0.958954526
V23	0.999469323	1.0001	0.99953761	0.999517464
V24	0.992	0.9873	0.99192508	0.991908823
V25	1.05	1.0536	1.05000332	1.050003541
V27	0.968	0.9732	0.96790403	0.967872613
V28	0.961568103	0.9598	0.96144365	0.961423985
V29	0.963216333	0.9696	0.96321247	0.963208882
V30	0.985332612	0.9891	0.98532212	0.985320696
V34	0.984	0.9922	0.9846821	0.984681453
V36	0.98	0.9866	0.98052999	0.980530005
V37	0.990661352	0.9967	0.99066773	0.990669113
V40	0.97	0.9747	0.97001014	0.970008582
V42	0.985	0.9796	0.98493382	0.984915786
V44	0.984436022	0.9748	0.98378291	0.983772111
V45	0.986382562	0.9905	0.98638227	0.986382139
V46	1.005	1.0054	1.00498386	1.005004244
V49	1.025	1.0334	1.02494218	1.024973998
V51	0.966876693	0.9762	0.96788721	0.967916688
V53	0.9459829	0.9407	0.94598246	0.945982509
V54	0.955	0.9515	0.95498754	0.954995267
V56	0.954	0.9511	0.95397181	0.953979339
V57	0.970582529	0.9646	0.97031236	0.970337061
V59	0.985	0.9915	0.9849461	0.984928541
V62	0.998	0.9923	0.99799786	0.997997814
V63	0.968737013	0.9648	0.96863925	0.968639248
V64	0.983738598	0.9782	0.98364967	0.983650588
V68	1.00324942	0.9981	1.00294887	1.002951107

V69	1.035	1.0435	1.03500487	1.035004712
V70	0.984	0.9887	0.98411157	0.984112084
V71	0.986844527	0.9886	0.98715507	0.987155499
V73	0.991	0.9973	0.99157682	0.99157786
V75	0.967331885	0.9578	0.96723838	0.967237901
V76	0.943	0.9415	0.94278286	0.942782183
V77	1.006	1.0122	1.00600075	1.006000823
V80	1.04	1.0495	1.04000262	1.040003152
V82	0.988545249	0.9897	0.98854571	0.988545768
V85	0.985	0.9763	0.98472478	0.984724801
V86	0.986690746	0.9875	0.98669053	0.986690534
V91	0.98	0.9868	0.98057786	0.980577872
V92	0.99	0.9806	0.98986926	0.989869266
V100	1.017	1.0221	1.01700054	1.017000551
V101	0.991419613	0.9859	0.99141947	0.991419473
V102	0.989130815	0.9915	0.98896194	0.988961944
V103	1.01	1.0141	1.00999635	1.009996388
V105	0.965	0.9742	0.96499557	0.964995617
V107	0.952	0.9527	0.95247693	0.952476968
V110	0.973	0.9825	0.97304885	0.973048888
V111	0.98	0.9782	0.97994201	0.979942036
V112	0.975	0.9667	0.97459342	0.974593449
V113	0.993	0.9858	0.99299812	0.99299765
V114	0.960093071	0.9645	0.96009345	0.960093415
P3	-0.39	-0.3849	-0.3840976	-0.38408919
P4	-0.39	-0.3958	-0.3959489	-0.39595604
P8	-0.28	-0.2824	-0.2819386	-0.28184678
P9	0	0.0000	1.408E-08	1.47155E-08
P12	0.38	0.3848	0.38299637	0.38302114
P13	-0.34	-0.3381	-0.3384861	-0.33843813
P15	-0.9	-0.8950	-0.8998529	-0.89984028
P16	-0.25	-0.2480	-0.2492553	-0.24923813
P19	-0.45	-0.4509	-0.4533844	-0.45337008
P20	-0.18	-0.1776	-0.1802247	-0.18021072
P24	-0.13	-0.1323	-0.132969	-0.13280696
P25	2.2	2.2435	2.23595636	2.235922409

P30	0	0.0000	-7.214E-07	-6.2478E-07
P31	-0.36	-0.3664	-0.3598686	-0.35991717
P33	-0.23	-0.2314	-0.2330865	-0.23316604
P35	-0.33	-0.3295	-0.3303	-0.33029872
P36	-0.31	-0.3044	-0.3051682	-0.30513725
P38	0	0.0000	1.7182E-07	1.16931E-07
P42	-0.96	-0.9690	-0.9679599	-0.96802037
P44	-0.16	-0.1581	-0.1587405	-0.15894085
P46	-0.09	-0.0884	-0.0884894	-0.08828365
P47	-0.34	-0.3455	-0.3441168	-0.34387558
P49	1.17	1.1639	1.16153746	1.161932867
P52	-0.18	-0.1806	-0.1815573	-0.18155875
P53	-0.23	-0.2321	-0.2282952	-0.22822852
P54	-0.65	-0.6479	-0.6489561	-0.64904261
P55	-0.63	-0.6404	-0.6388482	-0.63911379
P61	1.6	1.5911	1.58172873	1.583816823
P63	0	0.0000	-2.378E-06	-2.1432E-06
P64	0	0.0000	-1.575E-06	-1.1058E-06
P66	3.53	3.5670	3.55364862	3.554875959
P68	0	0.0000	6.191E-07	2.22295E-09
P70	-0.66	-0.6498	-0.6517482	-0.65199903
P71	0	0.0000	-2.088E-07	-2.2622E-07
P77	-0.61	-0.6014	-0.6091626	-0.60914211
P79	-0.39	-0.3952	-0.3938705	-0.39386758
P81	0	0.0000	2.5623E-07	-2.7322E-07
P83	-0.2	-0.2017	-0.1918489	-0.19184899
P85	-0.24	-0.2384	-0.2308309	-0.23083087
P86	-0.21	-0.2101	-0.2067757	-0.20677558
P89	6.07	5.9527	5.96430865	5.96430866
P90	-1.63	-1.6414	-1.6417902	-1.64179017
P92	-0.65	-0.6440	-0.6356514	-0.63565144
P96	-0.38	-0.3754	-0.3797882	-0.37978016
P97	-0.15	-0.1484	-0.1504278	-0.15042242
P98	-0.34	-0.3426	-0.3399183	-0.33991887
P99	-0.42	-0.4243	-0.421062	-0.42106051
P102	-0.05	-0.0491	-0.0429613	-0.04296135

P104	-0.38	-0.3732	-0.3774341	-0.37743418
P105	-0.31	-0.3093	-0.3107134	-0.31071343
P110	-0.39	-0.3969	-0.3969817	-0.39698167
P111	0.36	0.3566	0.35634878	0.356348766
P112	-0.68	-0.6720	-0.6729183	-0.67291832
P116	-1.84	-1.8683	-1.8438837	-1.84675384
P117	-0.2	-0.1997	-0.2000346	-0.20002209
P118	-0.33	-0.3321	-0.3288955	-0.32889575
Q3	-0.1	-0.1028	-0.1043564	-0.10436042
Q4	-0.270096466	-0.2755	-0.2763918	-0.27638445
Q8	0.631384439	0.6283	0.63581633	0.635827099
Q9	0	0.0000	-3.42E-07	-4.8831E-07
Q12	0.812917365	0.8385	0.83130504	0.831343991
Q13	-0.16	-0.1639	-0.1639393	-0.16395312
Q15	-0.228395456	-0.2335	-0.2319801	-0.23197756
Q16	-0.1	-0.1035	-0.1084827	-0.10845489
Q19	-0.392741715	-0.4078	-0.4049678	-0.40498208
Q20	-0.03	-0.0301	-0.0269262	-0.02694943
Q24	-0.149076212	-0.1525	-0.154313	-0.15432349
Q25	0.500432568	0.5166	0.50358234	0.503147211
Q30	0	0.0000	1.3562E-06	1.59676E-06
Q31	0.055860144	0.0579	0.05435405	0.054404719
Q33	-0.09	-0.0883	-0.0886497	-0.08861808
Q35	-0.09	-0.0909	-0.0890648	-0.08905657
Q36	-0.092749214	-0.0959	-0.0922731	-0.0922752
Q38	0	0.0000	2.8073E-06	3.57179E-06
Q42	0.180295754	0.1818	0.18186629	0.181659402
Q44	-0.08	-0.0780	-0.0876561	-0.08763267
Q46	-0.150294573	-0.1522	-0.1520082	-0.15219761
Q47	0	0.0000	5.4434E-07	5.64438E-07
Q49	0.8584513	0.8283	0.83784656	0.838067648
Q52	-0.05	-0.0487	-0.0501026	-0.05010048
Q53	-0.11	-0.1082	-0.1146105	-0.11481578
Q54	-0.280989355	-0.2778	-0.2784348	-0.27843489
Q55	-0.173358284	-0.1737	-0.172563	-0.17245333
Q61	-0.403940115	-0.3930	-0.3942663	-0.39544518

Q63	0	0.0000	1.1941E-06	1.17698E-06
Q64	0	0.0000	-6.127E-07	-6.273E-07
Q66	-0.199565866	-0.2026	-0.1936293	-0.19312353
Q68	0	0.0000	-5.715E-07	-5.9779E-07
Q70	-0.103307078	-0.1048	-0.1061837	-0.10612534
Q71	0	0.0000	2.4275E-07	2.47417E-07
Q77	-0.158296357	-0.1630	-0.1566779	-0.15669018
Q79	-0.32	-0.3166	-0.3162141	-0.31621602
Q81	0	0.0000	-7.127E-07	-7.4995E-07
Q83	-0.1	-0.0972	-0.0998173	-0.09981702
Q85	-0.206065892	-0.2070	-0.2096447	-0.20964475
Q86	-0.1	-0.0989	-0.098785	-0.09878517
Q89	-0.059049547	-0.0584	-0.06368	-0.06367997
Q90	0.173084075	0.1751	0.17321787	0.173217874
Q92	-0.239562482	-0.2305	-0.2382892	-0.23828921
Q96	-0.15	-0.1482	-0.1497609	-0.14976575
Q97	-0.09	-0.0931	-0.0904428	-0.09044608
Q98	-0.08	-0.0825	-0.0815931	-0.08159259
Q99	-0.175355895	-0.1684	-0.1710665	-0.1710677
Q102	-0.03	-0.0298	-0.0341733	-0.0341733
Q104	-0.226117482	-0.2255	-0.2253639	-0.22536369
Q105	-0.443345253	-0.4472	-0.4445844	-0.44458436
Q110	-0.297191664	-0.2865	-0.2846045	-0.2846045
Q111	-0.018438248	-0.0186	-0.0188661	-0.01886616
Q112	0.285116934	0.2812	0.27487421	0.274874165
Q116	0.513224772	0.4975	0.4926167	0.492490378
Q117	-0.08	-0.0800	-0.0840637	-0.08404455
Q118	-0.15	-0.1544	-0.1543617	-0.15436146
P1-2	-0.123528125	-0.1248	-0.1229186	-0.12289885
P3-5	-0.681105078	-0.6751	-0.679446	-0.67946399
P5-6	0.884704165	0.8771	0.8824895	0.882563257
P6-7	0.355399315	0.3614	0.36354086	0.363569105
P9-10	-4.452546498	-4.4367	-4.4523162	-4.45226091
P4-11	0.642297451	0.6419	0.64244178	0.642510942
P5-11	0.772246647	0.7812	0.77318693	0.773265878
P2-12	-0.324504197	-0.3309	-0.3245694	-0.32453722

P7-12	0.164798295	0.1617	0.16343323	0.163503507
P12-14	0.183143551	0.1837	0.18644732	0.186493549
P14-15	0.042380694	0.0422	0.040133	0.040344582
P17-18	0.802701822	0.8065	0.80272607	0.802699564
P21-22	-0.428374006	-0.4366	-0.4284803	-0.42847959
P23-24	0.08283672	0.0828	0.08160305	0.081943869
P28-29	0.156550728	0.1557	0.15394829	0.154079851
P30-17	2.311868411	2.2708	2.3070221	2.307744202
P17-31	0.147659266	0.1473	0.14765572	0.147642266
P23-32	0.929820076	0.9357	0.92103131	0.920981783
P34-36	0.302461705	0.2981	0.29722874	0.297338361
P37-40	0.440211114	0.4325	0.44019478	0.440194835
P39-40	0.269168157	0.2706	0.26912723	0.269125858
P40-41	0.154475226	0.1569	0.15766713	0.157426717
P43-44	-0.16592962	-0.1655	-0.1653873	-0.16613567
P34-43	0.014136648	0.0140	0.01454512	0.014179963
P46-48	-0.147609248	-0.1475	-0.146949	-0.14705096
P45-49	-0.497001544	-0.4872	-0.4969793	-0.49696554
P52-53	0.103713625	0.1033	0.10195785	0.101919507
P54-55	0.070734433	0.0714	0.07239778	0.072334894
P56-57	-0.229881793	-0.2260	-0.2275906	-0.22745168
P50-57	0.358754309	0.3629	0.36604185	0.365857767
P51-58	0.187862864	0.1886	0.18766202	0.187644735
P59-60	-0.433167199	-0.4372	-0.4382019	-0.43772948
P60-62	-0.098721958	-0.1004	-0.1018049	-0.09975433
P64-65	-1.827912937	-1.8604	-1.8368359	-1.83264544
P62-67	-0.243031519	-0.2387	-0.242944	-0.24287679
P65-68	0.141820054	0.1442	0.15124403	0.164176379
P47-69	-0.559400905	-0.5525	-0.5609355	-0.5597219
P71-72	0.106022225	0.1072	0.11109139	0.110877539
P71-73	0.060124025	0.0592	0.06113918	0.061313512
P69-75	1.100089657	1.1016	1.10016906	1.10017035
P74-75	-0.519885769	-0.5134	-0.5144345	-0.51450541
P76-77	-0.611500266	-0.6192	-0.6122252	-0.6122211
P78-79	-0.2568381	-0.2595	-0.2540396	-0.25403174
P81-80	-0.442019494	-0.4435	-0.4401191	-0.43624614

P77-82	-0.030254617	-0.0304	-0.0302433	-0.030243
P84-85	-0.36346101	-0.3682	-0.3555686	-0.35556863
P86-87	-0.039469498	-0.0392	-0.0395534	-0.03955339
P85-88	-0.503924269	-0.4945	-0.4910109	-0.49101089
P91-92	-0.085959829	-0.0850	-0.0761476	-0.07614759
P92-93	0.57623491	0.5728	0.56440509	0.564405085
P94-95	0.408609716	0.4122	0.40861646	0.408616483
P82-96	-0.09944337	-0.0979	-0.0994476	-0.09944765
P92-100	0.314970529	0.3211	0.311331	0.311331016
P95-96	-0.013762779	-0.0136	-0.0137761	-0.01377615
P98-100	-0.05256544	-0.0517	-0.0525408	-0.05254425
P99-100	-0.22648416	-0.2226	-0.2272168	-0.22721896
P100-101	-0.167426061	-0.1698	-0.1674568	-0.16745674
P101-102	-0.38979699	-0.3877	-0.3860451	-0.38604516
P100-106	0.603611779	0.5930	0.60360703	0.60360707
P105-108	0.239651084	0.2396	0.24100152	0.241001546
P108-109	0.217745778	0.2154	0.21515967	0.215159666
P109-110	0.13708691	0.1389	0.13875572	0.138755714
P17-113	0.020563362	0.0206	0.02054279	0.020536269
P27-115	0.207237411	0.2104	0.21410561	0.214282071
P114-115	0.013577432	0.0138	0.01357769	0.013577411
P12-117	0.201525499	0.2001	0.20158087	0.20156812
P75-118	0.40214829	0.3946	0.40256773	0.402561967
P76-118	-0.068499734	-0.0676	-0.0699939	-0.06998785
P12-11	-0.341478814	-0.3444	-0.3401186	-0.34016858
P13-11	-0.347681441	-0.3450	-0.3458089	-0.34591274
P15-13	-0.007670257	-0.0078	-0.0073123	-0.00746384
P17-15	1.054390221	1.0682	1.0584196	1.058384856
P19-18	-0.193095316	-0.1964	-0.1964263	-0.19648146
P21-20	0.288374006	0.2920	0.28934007	0.289353951
P22-21	0.432557451	0.4249	0.43265474	0.432654078
P23-22	0.542973375	0.5341	0.53385098	0.533849689
P25-23	1.667644778	1.6958	1.69286722	1.693148202
P25-26	-0.902890776	-0.9028	-0.8921533	-0.89243227
P28-27	-0.326550728	-0.3295	-0.3280671	-0.32818521
P30-8	-0.738054104	-0.7443	-0.7344564	-0.73489925

P31-29	0.084315937	0.0848	0.07857385	0.078577051
P34-19	0.036471224	0.0360	0.03595248	0.03577572
P37-35	0.339873984	0.3413	0.34053116	0.340388918
P37-33	0.158611428	0.1581	0.16012074	0.159939389
P38-30	-0.620909665	-0.6122	-0.6209627	-0.62093206
P42-40	0.119295261	0.1197	0.11580938	0.11640741
P42-41	0.218109223	0.2177	0.21455503	0.215274693
P47-46	0.314752951	0.3201	0.31389206	0.313560579
P49-42	0.680367349	0.6887	0.68087834	0.681636816
P49-42	0.680367349	0.6755	0.68087834	0.681636816
P49-48	0.351094767	0.3514	0.35363002	0.353206926
P50-49	-0.528754309	-0.5205	-0.5246806	-0.52416898
P51-49	-0.643450881	-0.6555	-0.6488894	-0.64836033
P52-51	-0.283713625	-0.2866	-0.2835151	-0.28347826
P54-49	-0.365752911	-0.3600	-0.3670975	-0.36680314
P56-54	-0.185170042	-0.1863	-0.1877784	-0.18783061
P56-55	0.214477318	0.2185	0.22045997	0.220151218
P58-56	0.066856666	0.0661	0.06482885	0.064805588
P59-54	0.30904078	0.3031	0.30970686	0.310348953
P59-56	0.300732162	0.3039	0.28740369	0.287954497
P61-59	0.526390965	0.5182	0.53022223	0.529611364
P61-60	1.124055258	1.1018	1.11087938	1.109242149
P59-63	-1.517714768	-1.5040	-1.5270158	-1.52466586
P64-63	1.522539323	1.5402	1.53189403	1.529531427
P66-49	1.352244387	1.3473	1.3610567	1.361474731
P66-62	0.379310181	0.3833	0.37963762	0.379457303
P66-65	-0.085406244	-0.0871	-0.0805385	-0.07965505
P67-66	-0.524990569	-0.5210	-0.5258031	-0.52549644
P69-49	0.487816757	0.4913	0.48894098	0.487653667
P69-68	1.258025073	1.2432	1.25437729	1.248192059
P70-24	0.062162419	0.0617	0.06542366	0.065178554
P72-24	-0.01449943	-0.0148	-0.0135591	-0.0138172
P74-70	-0.160114231	-0.1601	-0.1587177	-0.15874445
P77-69	-0.610491165	-0.6042	-0.6104758	-0.61047762
P77-75	0.354110617	0.3483	0.35417632	0.354176422
P80-79	0.655046824	0.6666	0.65612832	0.656117349

P83-82	0.475568976	0.4827	0.47554434	0.475544296
P86-85	-0.170530502	-0.1726	-0.1672224	-0.16722218
P87-86	0.04	0.0398	0.0400839	0.040083938
P89-88	1.003279652	1.0074	0.98824584	0.988245789
P90-89	-1.079341158	-1.0881	-0.5592737	-0.55927371
P94-92	-0.507485178	-0.5024	-0.5013807	-0.50138066
P96-94	-0.196596208	-0.1949	-0.1965915	-0.1965915
P98-80	-0.28743456	-0.2837	-0.2873776	-0.28737462
P100-94	-0.038663425	-0.0390	-0.0386302	-0.03863033
P97-96	0.111808416	0.1119	0.11160238	0.1115998
P103-100	-1.194020311	-1.1903	-1.1940186	-1.19401858
P105-104	-0.48334401	-0.4854	-0.4857842	-0.48578419
P106-105	-0.088482273	-0.0875	-0.0869399	-0.08693987
P107-106	-0.236527075	-0.2379	-0.2392943	-0.23929433
P109-108	-0.21708691	-0.2143	-0.2145067	-0.21450668
P113-32	-0.03947691	-0.0390	-0.0365267	-0.03656901
P114-32	-0.093577432	-0.0934	-0.0902696	-0.09019816
P116-68	-1.84	-1.8256	-1.8438837	-1.84675384
Q1-2	-0.130411997	-0.1267	-0.1304015	-0.13041107
Q3-5	-0.144888792	-0.1483	-0.1459954	-0.14595896
Q5-6	0.041064452	0.0415	0.04592239	0.045817829
Q6-7	-0.047715133	-0.0493	-0.0480358	-0.04808303
Q9-10	-0.244289062	-0.2443	-0.2425919	-0.2426378
Q4-11	-0.002176992	-0.0021	-0.0041461	-0.00424814
Q5-11	0.029659816	0.0290	0.02814946	0.028034349
Q2-12	-0.200063646	-0.2066	-0.2000479	-0.20010054
Q7-12	-0.065051818	-0.0664	-0.0721713	-0.07221846
Q12-14	0.026229983	0.0261	0.03213612	0.032091629
Q14-15	0.031405191	0.0323	0.03046026	0.030426492
Q17-18	0.247630295	0.2400	0.24786131	0.247949045
Q21-22	-0.020994276	-0.0214	-0.0203466	-0.02033263
Q23-24	0.10418757	0.1059	0.10743761	0.107263771
Q28-29	-0.065678558	-0.0639	-0.0662909	-0.06648396
Q30-17	0.929695148	0.9583	0.9290207	0.929056172
Q17-31	0.115217538	0.1136	0.11525753	0.115273984
Q23-32	0.050542069	0.0496	0.05159764	0.051465171

Q34-36	0.046954856	0.0486	0.05428103	0.05422218
Q37-40	-0.036750469	-0.0380	-0.0367675	-0.03675051
Q39-40	-0.087019243	-0.0899	-0.0872165	-0.08721149
Q40-41	0.011925986	0.0116	0.01145516	0.01164443
Q43-44	-0.013301855	-0.0136	-0.0131444	-0.01294245
Q34-43	0.016334408	0.0169	0.02367562	0.023853385
Q46-48	-0.058307704	-0.0561	-0.060228	-0.06033321
Q45-49	-0.020829791	-0.0209	-0.0205232	-0.0207058
Q52-53	0.019943001	0.0198	0.02463162	0.02477043
Q54-55	0.014568186	0.0147	0.01470017	0.014729945
Q56-57	-0.091035551	-0.0938	-0.0894898	-0.08971475
Q50-57	0.091409704	0.0878	0.09515967	0.095459416
Q51-58	0.031586831	0.0313	0.03668747	0.036797618
Q59-60	0.035742655	0.0364	0.03644158	0.036424888
Q60-62	-0.071142316	-0.0736	-0.0694125	-0.07048225
Q64-65	-0.664921362	-0.6705	-0.6597844	-0.6609263
Q62-67	-0.144140992	-0.1480	-0.1442397	-0.14435914
Q65-68	-0.22433052	-0.2251	-0.2188955	-0.21904753
Q47-69	0.116318079	0.1167	0.11671445	0.116250495
Q71-72	-0.009397096	-0.0094	-0.0105134	-0.01045489
Q71-73	-0.107384013	-0.1068	-0.1133915	-0.11343743
Q69-75	0.204853985	0.2124	0.20564717	0.205649559
Q74-75	-0.061904726	-0.0604	-0.0621704	-0.06215433
Q76-77	-0.210400076	-0.2104	-0.2114292	-0.21143543
Q78-79	-0.183717054	-0.1824	-0.1865359	-0.18654213
Q81-80	0.755392049	0.7379	0.74940614	0.749287989
Q77-82	0.175545436	0.1758	0.1755449	0.175544965
Q84-85	0.089918713	0.0895	0.08680768	0.086807783
Q86-87	-0.150909665	-0.1557	-0.1508896	-0.15088956
Q85-88	0.075996578	0.0731	0.07409224	0.074092302
Q91-92	-0.066257564	-0.0663	-0.0639231	-0.06392309
Q92-93	-0.11658282	-0.1143	-0.1065626	-0.10656261
Q94-95	0.090142431	0.0934	0.09016869	0.090168582
Q82-96	-0.065687024	-0.0648	-0.0656874	-0.06568729
Q92-100	-0.165322796	-0.1713	-0.1653127	-0.16531272
Q95-96	-0.216886523	-0.2188	-0.2169131	-0.21691377

Q98-100	0.024288412	0.0238	0.02369308	0.023695933
Q99-100	-0.045940966	-0.0467	-0.0428443	-0.04284255
Q100-101	0.228998132	0.2323	0.22901141	0.229011457
Q101-102	0.101278388	0.1027	0.10177742	0.101777427
Q100-106	0.094759835	0.0913	0.09463666	0.094636492
Q105-108	-0.111284593	-0.1074	-0.1091425	-0.10914243
Q108-109	-0.109223146	-0.1134	-0.111898	-0.11189793
Q109-110	-0.133929253	-0.1392	-0.1362339	-0.13623375
Q17-113	0.059005991	0.0566	0.05901306	0.059015297
Q27-115	0.050603851	0.0516	0.04793435	0.047487317
Q114-115	0.002207459	0.0021	0.00220587	0.002207574
Q12-117	0.051972517	0.0529	0.05614549	0.056124884
Q75-118	0.235876598	0.2421	0.23909102	0.239094219
Q76-118	-0.096918915	-0.0995	-0.0956624	-0.09566596
Q12-11	0.351306354	0.3379	0.34555594	0.345586238
Q13-11	-0.121563874	-0.1259	-0.1256738	-0.12565152
Q15-13	-0.020399604	-0.0202	-0.0205521	-0.02051534
Q17-15	0.25219607	0.2547	0.25501594	0.255036553
Q19-18	-0.175472634	-0.1723	-0.1772121	-0.17717795
Q21-20	-0.059005724	-0.0578	-0.0544688	-0.05449861
Q22-21	0.017578578	0.0179	0.016832	0.016819015
Q23-22	0.076857162	0.0749	0.07126435	0.071220856
Q25-23	0.386264159	0.3793	0.38364187	0.383885348
Q25-26	-0.186396463	-0.1919	-0.1812523	-0.18213006
Q28-27	-0.004321442	-0.0044	-0.0042431	-0.00407764
Q30-8	-0.754234945	-0.7834	-0.7558525	-0.75580393
Q31-29	0.079204522	0.0803	0.0809428	0.080974377
Q34-19	0.045965645	0.0452	0.04943344	0.049498648
Q37-35	0.124274652	0.1219	0.1153555	0.115405497
Q37-33	0.074589889	0.0768	0.07467203	0.074727399
Q38-30	-0.559788328	-0.5403	-0.5602552	-0.56046917
Q42-40	0.023016931	0.0221	0.02358842	0.023330967
Q42-41	0.05238221	0.0527	0.052961	0.052686547
Q47-46	-0.007948074	-0.0080	-0.0083325	-0.00812451
Q49-42	0.003701708	0.0037	0.00373308	0.003882726
Q49-42	0.003701708	0.0038	0.00373308	0.003882726

Q49-48	-0.039251602	-0.0388	-0.0473186	-0.04746277
Q50-49	-0.131409704	-0.1303	-0.1253115	-0.12520157
Q51-49	-0.174043655	-0.1775	-0.1643228	-0.16459334
Q52-51	-0.069943001	-0.0721	-0.0747342	-0.07487091
Q54-49	-0.155988734	-0.1577	-0.1553441	-0.15553527
Q56-54	-0.049750398	-0.0496	-0.0505634	-0.0505686
Q56-55	0.055660332	0.0538	0.05518879	0.055345823
Q58-56	0.015329924	0.0153	0.02269584	0.022847236
Q59-54	0.042569534	0.0432	0.04229447	0.042093221
Q59-56	0.011295402	0.0113	0.00960804	0.009373792
Q61-59	-0.046257928	-0.0476	-0.0461338	-0.04610287
Q61-60	-0.082273513	-0.0831	-0.080852	-0.07988246
Q59-63	-0.570248383	-0.5856	-0.5684335	-0.56900939
Q64-63	0.525058422	0.5083	0.52493478	0.525109911
Q66-49	0.083248828	0.0853	0.08322471	0.083153581
Q66-62	0.146757189	0.1514	0.14672874	0.146893864
Q66-65	-0.705545993	-0.6965	-0.6993397	-0.69908407
Q67-66	-0.191470858	-0.1917	-0.1912043	-0.19145621
Q69-49	-0.12058744	-0.1240	-0.1205511	-0.12047055
Q69-68	-1.036430373	-1.0038	-1.0274526	-1.02778183
Q70-24	-0.068017972	-0.0701	-0.0675151	-0.06748048
Q72-24	-0.079823677	-0.0776	-0.0781969	-0.07805826
Q74-70	-0.154215682	-0.1599	-0.1556874	-0.15568808
Q77-69	-0.138028334	-0.1372	-0.1380761	-0.13807317
Q77-75	0.073762897	0.0760	0.07421393	0.074216679
Q80-79	0.310833333	0.2989	0.30988586	0.309893203
Q83-82	-0.269927123	-0.2697	-0.2699178	-0.26991796
Q86-85	0.050909665	0.0497	0.05210462	0.052104389
Q87-86	0.110216074	0.1112	0.11019638	0.110196376
Q89-88	0.076986325	0.0756	0.07822935	0.078229328
Q90-89	0.070712826	0.0725	0.05787354	0.057873524
Q94-92	0.159054268	0.1633	0.15742152	0.157421548
Q96-94	0.07976965	0.0799	0.07977373	0.07977419
Q98-80	-0.104288412	-0.1038	-0.1052862	-0.10528852
Q100-94	0.458060865	0.4462	0.45805898	0.458058997
Q97-96	0.181870386	0.1884	0.18164369	0.181645832

Q103-100	0.243578272	0.2370	0.24349638	0.243496721
Q105-104	-0.026060737	-0.0260	-0.0263696	-0.02636952
Q106-105	-0.051516521	-0.0506	-0.0513304	-0.05133029
Q107-106	0.005505287	0.0053	0.00877211	0.008772056
Q109-108	0.103929253	0.1055	0.10659001	0.106589984
Q113-32	0.133988283	0.1336	0.13247089	0.132500794
Q114-32	-0.032207459	-0.0322	-0.034723	-0.03468026
Q116-68	0.513224772	0.4934	0.4926167	0.492490378
Va2	-18.48745255	-18.48745255	-18.492305	-18.4828603
Va9	-1.705310551	-1.705310551	-1.7070876	-1.69797525
Va17	-16.00477855	-16.00477855	-15.98463	-15.9783085
Va25	-1.820161215	-1.820161215	-1.7926733	-1.78447889
Va30	-10.96624656	-10.96624656	-10.956624	-10.9487154
Va37	-18.0333208	-18.0333208	-18.024633	-18.0220604
Va45	-14.2274162	-14.2274162	-14.24886	-14.2243209
Va53	-15.56385127	-15.56385127	-15.610643	-15.5807395
Va62	-6.495166051	-6.495166051	-6.4755615	-6.44759246
Va69	0	0	0	0
Va77	-3.249363242	-3.249363242	-3.249179	-3.24919413
Va82	-2.728262824	-2.728262824	-2.7281357	-2.72815222
Va86	1.186172588	1.186172588	1.156519	1.156505384
Va94	-1.317794965	-1.317794965	-1.3176731	-1.31768912
Va100	-1.941158013	-1.941158013	-1.9409243	-1.94094067
Va114	-15.27359355	-15.27359355	-15.280319	-15.2731212
Vpmu2	0.971393	0.971393	0.97139218	0.971392149
Vpmu9	1.042918	1.042918	1.0429192	1.042919526
Vpmu17	0.995089	0.995089	0.99508668	0.995086219
Vpmu25	1.05	1.05	1.05000332	1.050003541
Vpmu30	0.985333	0.985333	0.98532212	0.985320696
Vpmu37	0.990661	0.990661	0.99066773	0.990669113
Vpmu45	0.986383	0.986383	0.98638227	0.986382139
Vpmu53	0.945983	0.945983	0.94598246	0.945982509
Vpmu62	0.998	0.998	0.99799786	0.997997814
Vpmu69	1.035	1.035	1.03500487	1.035004712
Vpmu77	1.006	1.006	1.00600075	1.006000823
Vpmu82	0.988545	0.988545	0.98854571	0.988545768

Vpmu86	0.986691	0.986691	0.98669053	0.986690534
Vpmu94	0.98983	0.98983	0.98983053	0.989830548
Vpmu100	1.017	1.017	1.01700054	1.017000551
Vpmu114	0.960093	0.960093	0.96009345	0.960093415
IFR9-10	-4.260453696	-4.260453696	-4.2602662	-4.26026771
IFR2-12	-0.251513057	-0.251513057	-0.2515565	-0.25155757
IFR17-18	0.706783391	0.706783391	0.7069065	0.706907864
IFR25-27	1.357119198	1.357119198	1.35725078	1.357257427
IFR30-17	2.123948595	2.123948595	2.11950295	2.120407888
IFR17-31	0.110712066	0.110712066	0.1107513	0.110750592
IFR37-39	0.517728296	0.517728296	0.51771838	0.51771837
IFR37-40	0.434016438	0.434016438	0.43401879	0.434017518
IFR30-38	0.584507941	0.584507941	0.5845222	0.584492068
IFR45-46	-0.348110352	-0.348110352	-0.3481022	-0.34810129
IFR45-49	-0.483218273	-0.483218273	-0.483219	-0.48322179
IFR53-54	-0.113409299	-0.113409299	-0.1134159	-0.11341668
IFR62-66	-0.3504111	-0.3504111	-0.3508092	-0.35071714
IFR62-67	-0.225617692	-0.225617692	-0.2255784	-0.22558153
IFR69-70	1.047110606	1.047110606	1.04710009	1.047097613
IFR69-75	1.062888558	1.062888558	1.06296027	1.062961683
IFR69-77	0.601056846	0.601056846	0.60103889	0.601040738
IFR77-78	0.446789103	0.446789103	0.44679683	0.446796819
IFR77-80	-0.937325086	-0.937325086	-0.9373653	-0.93734696
IFR77-80	-0.428722119	-0.428722119	-0.9373653	-0.93734696
IFR77-82	-0.039916689	-0.039916689	-0.0399048	-0.03990459
IFR82-83	-0.488899655	-0.488899655	-0.4888739	-0.48887387
IFR86-87	-0.043159463	-0.043159463	-0.0431653	-0.04316532
IFR94-95	0.410604213	0.410604213	0.4106106	0.410610597
IFR82-96	-0.097318766	-0.097318766	-0.0973232	-0.09732317
IFR94-96	0.20214094	0.20214094	0.20213609	0.202136076
IFR94-100	0.054979735	0.054979735	0.05494499	0.054945245
IFR100-101	-0.17216014	-0.17216014	-0.1721898	-0.17218981
IFR100-103	1.203870882	1.203870882	1.20386462	1.203864617
IFR100-104	0.54859165	0.54859165	0.54856236	0.548562356
IFR100-106	0.590025152	0.590025152	0.59002474	0.590024742
IFR17-113	0.003514575	0.003514575	0.0035148	0.003514798

IFR114-115	0.013036606	0.013036606	0.0130366	0.013036597
IFI9-10	0.361182408	0.361182408	0.359681	0.359045776
IFI2-12	0.301256387	0.301256387	0.30128377	0.301283226
IFI17-18	-0.461618006	-0.461618006	-0.4616002	-0.46159983
IFI25-27	-0.32952397	-0.32952397	-0.3294694	-0.32946373
IFI30-17	-1.372639709	-1.372639709	-1.3706907	-1.37057468
IFI17-31	-0.152211386	-0.152211386	-0.1522104	-0.15221041
IFI37-39	-0.200182779	-0.200182779	-0.2001855	-0.20018551
IFI37-40	-0.102286223	-0.102286223	-0.1021983	-0.102195
IFI30-38	-0.309943346	-0.309943346	-0.3103712	-0.31051259
IFI45-46	0.125568092	0.125568092	0.1255824	0.125583801
IFI45-49	0.144304642	0.144304642	0.14417873	0.144147782
IFI53-54	0.092500918	0.092500918	0.09250073	0.092501024
IFI62-66	0.214004208	0.214004208	0.21384546	0.213837512
IFI62-67	0.171049485	0.171049485	0.17106107	0.171062261
IFI69-70	-0.155311199	-0.155311199	-0.1544896	-0.15448475
IFI69-75	-0.197926556	-0.197926556	-0.198692	-0.19869432
IFI69-77	-0.065537848	-0.065537848	-0.0655821	-0.06557938
IFI77-78	-0.091189857	-0.091189857	-0.0911576	-0.09115767
IFI77-80	0.425731955	0.425731955	0.42575446	0.425770137
IFI77-80	0.228899918	0.228899918	0.42575446	0.425770137
IFI77-82	-0.172513249	-0.172513249	-0.1725134	-0.17251341
IFI82-83	-0.223744063	-0.223744063	-0.2237357	-0.22373567
IFI86-87	0.152084392	0.152084392	0.15208467	0.152084666
IFI94-95	-0.100538153	-0.100538153	-0.1005639	-0.10056396
IFI82-96	0.071161124	0.071161124	0.07116143	0.071161378
IFI94-96	0.094601936	0.094601936	0.0946067	0.09460711
IFI94-100	0.509438519	0.509438519	0.50943778	0.509437768
IFI100-101	-0.219464579	-0.219464579	-0.2194772	-0.21947719
IFI100-103	0.177105375	0.177105375	0.17703154	0.177031526
IFI100-104	-0.123343499	-0.123343499	-0.1231799	-0.12317989
IFI100-106	-0.113226813	-0.113226813	-0.1131031	-0.11310315
IFI17-113	-0.062696454	-0.062696454	-0.0626965	-0.06269646
IFI114-115	-0.005943352	-0.005943352	-0.0059434	-0.00594335

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